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Punctual Graph Topological Space: Expansible and Non-Expansible Cases

Ekhlas S. Malalla [(1)]*,
Amir A. Mohammed [(2)],

[1,2]**Department of Mathematics, Collage of Education for Pure Science, University of Mosul, Mosul, Iraq**

***Corresponding author e-mail:**
ekhlas.23esp84@student.uomosul.edu.iq

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ABSTRACT

The relationship between graph theory and topological space is a multifaceted one, where graph theory can be used to represent topological spaces, facilitating their understanding and the solution of associated problems in an effective manner. Additionally, graphs can be generated from topological spaces, opening up new avenues for research in various fields. In this research, we will explore the relationship between graph theory and topological space, and seek to develop a theoretical framework that combines graph topological space with graph theory, focusing on its practical applications in various fields. The main objective of this paper is to present a new method for developing and applying graph topological space in various fields, including urban planning and neuroscience, which can contribute to improving the quality of life in modern societies. The results obtained indicate that this work can contribute to a better understanding of life models and provide assistance in solving some problems in daily life, opening up new avenues for research in various fields, and creating new doors for innovation and improvement in areas of life.

الفضاء التبولوجي البياني النقطي: الحالات القابلة للتوسيع وغير القابلة للتوسيع

اخلاص سليمان مال الله [(1)]، **عامر عبدالاله محمد** [(2)]
[(1، 2)] **قسم الرياضيات، كلية التربية للعلوم الصرفة، جامعة الموصل، موصل–العراق**

الخلاصة:

العلاقة بين نظرية الرســم البياني والفضـــاء التوبولوجي هي علاقة متعددة الجوانب، حيث يمكن استخدام الرسم البياني لتمثيل الفضـاءات التوبولوجية، مما يسهل من فهمها وحل المشـاكل المرتبطة بها. كما يمكن توليد الرسم البياني من الفضـاءات التوبولوجية. في هذا البحث، سنستكشف العلاقة بين نظرية الرسـم البياني والفضـاء التوبولوجي، وسـنطور إطارًا نظريًا يجمع بين الفضـاء التوبولوجي البياني ونظرية الرسم البياني. الهدف الرئيسي لهذا البحث هو تقديم طريقة جديدة في تطوير وتطبيق الفضاء التوبولوجي

البياني في مجالات مختلفة، بما في ذلك التخطيط العمراني وعلم الأعصــــاب. تشـــير النتائج التي تم الحصـول عليها إلى أن هذا العمل يمكن أن يسـاهم في فهم النماذج الحياتية بشـكل أفضـل، ويوفر دعمًا في حل بعض المشاكل في الحياة اليومية، مما يفتح أبوابًا جديدة للبحث في مجالات مختلفة.

الكلمات المفتاحية: التخطيط العمراني، علم الأعصاب، بيان المناقلة.

1-INTRODUCTION

Graph theory is a fundamental mathematical tool that is widely used in various practical applications, such as computer networks, electrical engineering, industry, and chemistry. It has also been used to study the topological properties of digital images and complex networks, such as social networks [1][2]. Previous studies have included topological models to understand the structure of graphs, like S. M. Amiri et al.'s study [3]. Recently, new studies have emerged showing the importance of topology in understanding complex networks and providing models to explain their behavior, especially in machine learning and distributed environments [4][5].Many problems that arise in different fields can be formulated as graph theory problems, highlighting the importance of this theory in solving complex problems. Historically, mathematics in general has emerged to meet the needs of practical applications. For example, geometry emerged to measure land and organize agriculture, while topology emerged to address engineering problems. Based on mathematical theories, one can use them to solve many complex problems in various fields. Researchers have used different relationships to link topological spaces with graphs, including relationships that connect topological spaces with graphs through vertices[6][7][8]. In this paper we denoted by τ^i the family of $i-open$ sets. We present our work in two sections. In the first section, Expansible Punctual Graph Topological Space, our method is to apply graph topological space to various life models, resulting in a deeper understanding of the internal structure of these models. The results indicate that this method can help improve the design of urban areas and understand the behavior of neurons, opening up new avenues in the field of graph topological space and its applications. In the second section, we presented a study on Non-Expansible Punctual Graph Topological Space, focusing on transition graphs as a key example. Now we present some basic definitions and illustrative examples for later use, as follows:

Definition 1.1[9] let W be a subset of a topological space $(\mathcal{N}, \tau)$ then W is i-open if there exists $H \in \tau /\{\mathcal{N}, \emptyset\}$ such that $W \subset cl\,(W \cap H)$.

Example 1.2 let $\mathcal{N}$=$\{d, e, f\}$, τ=$\{\emptyset, \{d\}, \{d, e\}, \mathcal{N}\}$ and let W=$\{d\}$ then W is i-open because $W \subset cl\,(W \cap \{d, e\}) = cl\,(\{d\}) = \mathcal{N}$.

Theorem 1.3 [9] All open sets in a topological space are i-open, but the converse is not always true.

Example 1.4 let $\mathcal{N}$=$\{k, z, c\}$, τ=$\{\emptyset, \{k\}, \{k, c\}, \mathcal{N}\}$ and let W=$\{k, z\}$ then W is i-open but it is not open**.**

Definition 1.5 [10] $(\mathcal{N}, \tau)$ is said to be T_0-space if any $a, e \in \mathcal{N}$ $(a \neq e)$, there exist $L \in \tau$ such that either $a \in L, e \notin L$ or $e \in L, a \notin L$.

Example 1.6 let $\mathcal{N}$=$\{k, z\}$, τ=$\{\emptyset, \{k\}, \mathcal{N}\}$, $(\mathcal{N}, \tau)$ is topological space.
$k, z \in \mathcal{N}$ $(k \neq z)$, $\exists\, \{k\} \in \mathcal{N}$ s.t $k \in \{k\}, z \notin \{k\}$. So we have $(\mathcal{N}, \tau)$ is T_0-space.

Definition 1.7 [10] $(\mathcal{N}, \tau)$ is said to be T_1-space if any $a, e \in \mathcal{N}$ $(a \neq e)$, there exist $K, L \in \tau$ such that $a \in L, e \notin L$ and $a \notin K, e \in K$.

Example 1.8 let $\mathcal{N}$=$\{f, p, d\}$, τ=$\{\emptyset, \{f\}, \{p\}, \{d\}, \{f, p\}, \{f, d\}, \{p, d\}, \mathcal{N}\}$, $(\mathcal{N}, \tau)$ is topological space.
$f, p \in \mathcal{N}$ $(f \neq p)$, $\exists\, \{p\}, \{f\} \in \tau$
s.t $p \in \{p\}, f \notin \{p\}, f \in \{f\}, p \notin \{f\}$.
$f, d \in \mathcal{N}$ $(f \neq d)$, $\exists\, \{d\}, \{f\} \in \tau$
s.t $d \in \{d\}, f \notin \{d\}, f \in \{f\}, d \notin \{f\}$.
$d, p \in \mathcal{N}$ $(d \neq p)$, $\exists\, \{p\}, \{d\} \in \tau$
s.t $p \in \{p\}, d \notin \{p\}, d \in \{d\}, p \notin \{d\}$.
So we have $(\mathcal{N}, \tau)$ is T_1-space.

Definition 1.9 [11]A graph is anordered pair $G = (N, E)$, where N is a set of vertices or nodes, and P is a set of edges or arcs or relashinships that connect pairs of vertices, such that each edge connects two vertices. A graph can be represented graphically using points and lines, where points represent vertices and lines represent edges.

Definition 1.10 [12] The transition graph IG is a graph associated with the original graph G, such that the vertices of IG represent the edges of the original graph G.

Example 1.11 Look at the graph $G = (N, E)$, as shown in the following

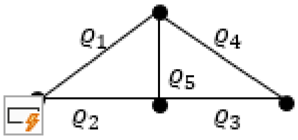
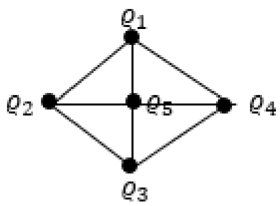


Fig.1… (G)

Then the transition graph IG of graph G is as follows:



(IG)

2-PUNCTUAL GRAPH TOPOLOGICAL SPACE

In this section, we define punctual graph topological space to prove our result (proposition 2.3)

Definition 2.1[13] Let $G = (N,)$ be a graph and let $I(\varrho)$ be the vertices associated with the edge ϱ, then $\tau_g = \{\cup S_{I(\varrho)} : \varrho \in E\}$ represented a topology on the set of vertices of the graph, where $S_{I(\varrho)}$ represent a topological basis for the topological space, and is defined as follows:

$S_{I(\varrho)} = \{\cap I(\varrho) : \varrho \in E\} \cup \{I(\varrho)\}$.

Example 2.2 Look at the graph $G = (N, E)$, as shown in the following

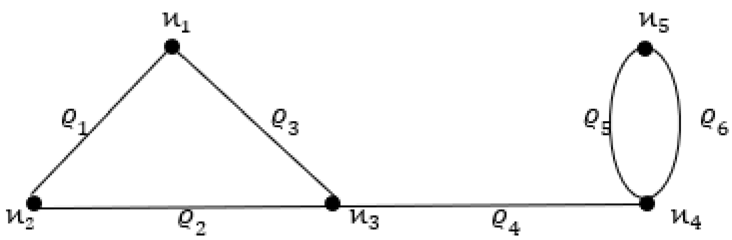


Fig.2

Let $\acute{G} = \{u_1, u_2, u_3, u_4, u_5\}$, hence
$I(\varrho_1) = \{u_1, u_2\}$, $I(\varrho_2) = \{u_2, u_3\}$, $I(\varrho_3) = \{u_1, u_3\}$, $I(\varrho_4) = \{u_3, u_4\}$, $I(\varrho_5) = \{u_4, u_5\}$, $I(\varrho_6) = \{u_4, u_5\}$
Then the sub basis $S_{I(\varrho)}$ is as follows:
$S_{I(\varrho)} = \{\emptyset, \{u_1, u_2\}, \{u_2, u_3\}, \{u_1, u_3\}, \{u_3, u_4\}, \{u_4, u_5\}, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}\}$
Therefore, the topology on the space $\acute{G}$ can be written as:
$\tau_g = \{\emptyset, \{u_1, u_2\}, \{u_2, u_3\}, \{u_1, u_3\}, \{u_3, u_4\}, \{u_4, u_5\}, \{u_1\}, \{u_2\}, \{u_3\}, \{u_4\}, \{u_1, u_4\}, \{u_2, u_4\}, \{u_1, u_2, u_3\}, \{u_1, u_2, u_4\}, \{u_2, u_3, u_4\}, \{u_1, u_3, u_4\}, \{u_1, u_4, u_5\}, \{u_2, u_4, u_5\}, \{u_3, u_4, u_5\}, \{u_1, u_2, u_3, u_4\}, \{u_1, u_2, u_4, u_5\}, \{u_2, u_3, u_4, u_5\}, \{u_1, u_3, u_4, u_5\}, \acute{G}\}$

Proposition 2.3 Let $C_n = (\mathrm{N}, \mathrm{E})$ be a cycle graph whose vertices set is $\mathrm{N} = \{u_1, u_2, u_3, \ldots, u_n\}$ where $n \geq 3$ Then the topological space generated by the cycle graph $C_n = (\mathrm{N}, \mathrm{E})$ is a discrete topological space.
Proof: Let $C_n = (\mathrm{N}, \mathrm{E})$ be a cycle graph is as in a Fig.3

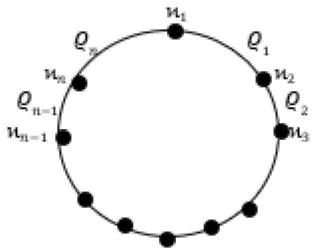


Fig.3

We have

$$I(\varrho_1) = \{u_1, u_2\},\ I(\varrho_2) = \{u_2, u_3\},\ I(\varrho_3) = \{u_3, u_4\},$$
$$\ldots, I(\varrho_{n-1}) = \{u_{n-1}, u_n\},\ I(\varrho_n) = \{u_n, u_1\}.$$

Then the sub-basis $S_{I(\varrho)}$ is as follows:
$S_{I(\varrho)} = \{\emptyset, \{u_1\}, \{u_2\}, \{u_3\}, \ldots \ldots, \{u_{n-1}\}, \{u_n\}, \{u_i, u_{i+1}\}_{i=1}^{n-1}\}$

Therefore, the topology generated by the cycle C_n is:
$\tau_g =$
$P(\mathrm{N}) = \{\emptyset, \{u_1\}, \{u_2\}, \{u_3\}, \ldots \ldots, \{u_{n-1}\}, \{u_n\}, \{u_i, u_{i+1}\}_{i=1}^{n-1}, \{u_i, u_{i+1}\} \cup_{i=1}^{n-1}, \acute{G}\}$

Hence, the topological space $(\tau_g, \acute{G})$ has been shown to be discrete.

3-EXPANSIBLE PUNCTUAL GRAPH TOPOLOGICAL SPACE

In this section, we define and apply this concept to various life models, including neurons and geographical areas, where these models have been transformed into expansible topological spaces, allowing them to be reconfigured based on a expansible topological space. The results obtained indicate that reconfiguring these models can lead to changes in their shapes. This study opens new doors in the field of graph topological spaces and their applications in many applied fields.

Definition 3.1 A punctual graph topological space $G = (N, E)$ is said to be expansible punctual graph topological space if $\tau_g \subsetneq \tau_g^i$.

Example 3.2 Look at the graph $G = (N, E)$, as shown in the following

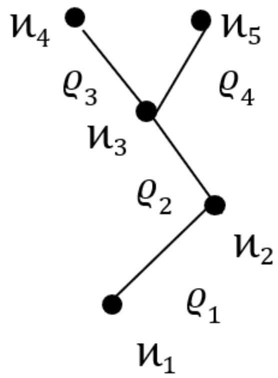


Fig.4

Let $\acute{G} = \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}$, hence

$I(\varrho_1) = \{\varkappa_1, \varkappa_2\}$, $I(\varrho_2) = \{\varkappa_2, \varkappa_3\}$, $I(\varrho_3) = \{\varkappa_3, \varkappa_4\}$, $I(\varrho_4) = \{\varkappa_3, \varkappa_5\}$,

Then the sub basis $S_{I(\varrho)}$ is as follows:

$S_{I(\varrho)}$= $\{\emptyset, \{\varkappa_1, \varkappa_2\}, \{\varkappa_2, \varkappa_3\}, \{\varkappa_3, \varkappa_4\}, \{\varkappa_3, \varkappa_5\}, \{\varkappa_2\}, \{\varkappa_3\}\}$

Therefore, the topology on the space $\acute{G}$ can be written as:

τ_g =$\{\emptyset, \{\varkappa_1, \varkappa_2\}, \{\varkappa_2, \varkappa_3\}, \{\varkappa_3, \varkappa_4\}, \{\varkappa_3, \varkappa_5\}, \{\varkappa_2\}, \{\varkappa_3\}, \{\varkappa_1, \varkappa_2, \varkappa_3\}, \{\varkappa_2, \varkappa_3, \varkappa_4\},$ $\{\varkappa_2, \varkappa_3, \varkappa_5\}, \{\varkappa_3, \varkappa_4, \varkappa_5\}, \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}, \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_5\}, \{\varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}, \acute{G}\}$

Now, we find τ_g^i={family of i −open sets}

So, we have

τ_g^i =$\{\emptyset, \{\varkappa_1, \varkappa_2\}, \{\varkappa_2, \varkappa_3\}, \{\varkappa_3, \varkappa_4\}, \{\varkappa_3, \varkappa_5\}, \{\varkappa_2\}, \{\varkappa_3\}, \{\varkappa_1, \varkappa_2, \varkappa_3\}, \{\varkappa_2, \varkappa_3, \varkappa_4\},$ $\{\varkappa_2, \varkappa_3, \varkappa_5\}, \{\varkappa_3, \varkappa_4, \varkappa_5\}, \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_4\}, \{\varkappa_1, \varkappa_2, \varkappa_3, \varkappa_5\}, \{\varkappa_2, \varkappa_3, \varkappa_4, \varkappa_5\}, \{\varkappa_1\},$

$\{\text{ℵ}_4\}, \{\text{ℵ}_1, \text{ℵ}_3\}$, $\{\text{ℵ}_1, \text{ℵ}_4\}$, $\{\text{ℵ}_2, \text{ℵ}_4\}, \{\text{ℵ}_1, \text{ℵ}_2, \text{ℵ}_4\}$, $\{\text{ℵ}_1, \text{ℵ}_3, \text{ℵ}_4\}$, $\{\text{ℵ}_1, \text{ℵ}_3, \text{ℵ}_5\}$, $\{\text{ℵ}_1, \text{ℵ}_3, \text{ℵ}_4, \text{ℵ}_5\}, \acute{G}\}$

Thus, we obtain $\tau_g \subsetneq \tau_g^i$, which implies that the punctual graph topological space is expansible topological space.

3.3 Improving Traffic Flow using Punctual Graph Topological Space

Within the framework of searching for innovative methods to improve urban planning and infrastructure, a study was conducted in a specific residential region consisting of streets and important elements such as a hospital, university, park, and Mosque. Where we transformed the region into a graph, and then generated a punctual graph topological space from a graph, which was found to be an expansible graph topological space. Our method aims to study the changes that occur in the region's planning when it is reconfigured based on the expansible graph topological space. The results obtained indicate that adding a new street to the region can lead to improved traffic flow and reduced congestion in the region. To illustrate these results, we present a practical example of how this approach can be applied, as follows:

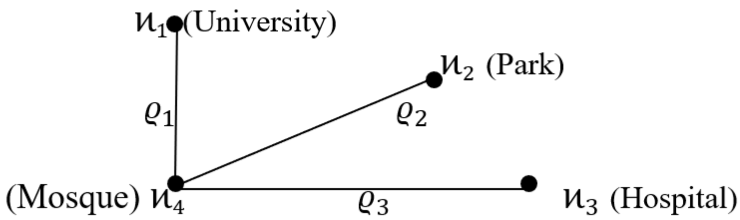


Fig.5

By using the graphical representation of residential region, we have

$I(\varrho_1) = \{\text{ℵ}_1, \text{ℵ}_4\}$, $I(\varrho_2) = \{\text{ℵ}_2, \text{ℵ}_4\}$, $I(\varrho_3) = \{\text{ℵ}_3, \text{ℵ}_4\}$

Then the sub basis $S_{I(\varrho)}$ is as follows:

$S_{I(\varrho)} = \{\emptyset, \{\text{ℵ}_1, \text{ℵ}_4\}, \{\text{ℵ}_2, \text{ℵ}_4\}, \{\text{ℵ}_3, \text{ℵ}_4\}, \{\text{ℵ}_4\}\}$

Therefore, the topology on the space $\acute{G}$ can be written as:

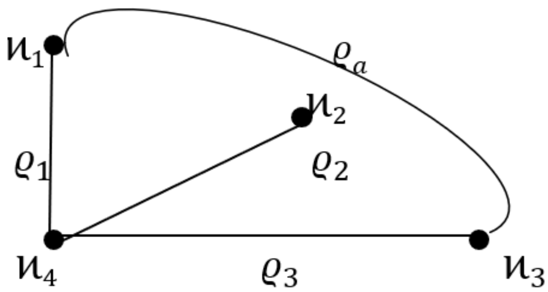
$\tau_g = \{\emptyset, \{\text{ℵ}_1, \text{ℵ}_4\}, \{\text{ℵ}_2, \text{ℵ}_4\}, \{\text{ℵ}_3, \text{ℵ}_4\}, \{\text{ℵ}_4\}, \{\text{ℵ}_1, \text{ℵ}_2, \text{ℵ}_4\}, \{\text{ℵ}_2, \text{ℵ}_3, \text{ℵ}_4\}, \{\text{ℵ}_1, \text{ℵ}_3, \text{ℵ}_4\}, \acute{G}\}$

Now, we proceed to find τ_g^i={family of i −open sets}

Hence

$\tau_g^i =$
$\{\emptyset, \{u_1, u_4\}, \{u_2, u_4\}, \{u_3, u_4\}, \{u_4\}, \{u_1, u_2, u_4\}, \{u_2, u_3, u_4\}, \{u_1, u_3, u_4\}, \{u_1\}, \{u_3\}, \{u_1, u_3\}, \acute{G}\}$

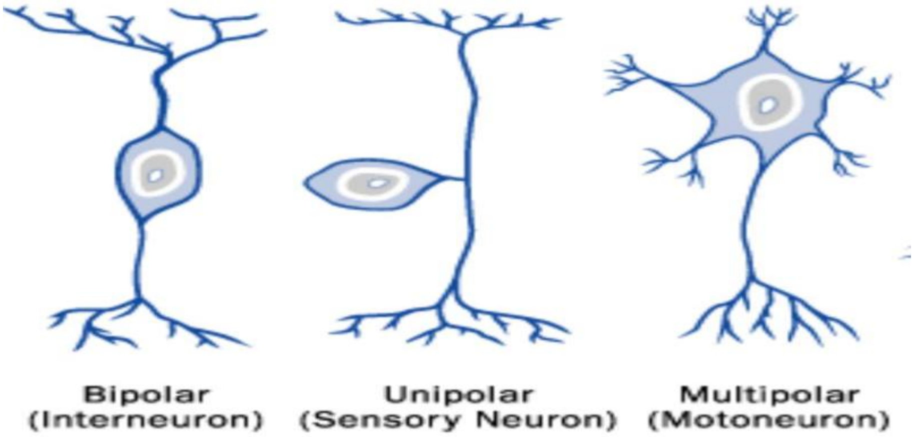
Based on the above, we conclude that the punctual graph topological space is an expansible space, enabling us to expand the original shape of the region. As a result, its shape will undergo a slight transformation when redrawn from the expansible punctual graph topological space, a new edge, denoted by the symbol ϱ_a, will be added, resulting in the following modified shape



This result provides a significant benefit in improving urban planning and infrastructure, as the punctual graph topological space can be used to determine the necessary changes to improve traffic flow and reduce congestion in urban areas.

3.4 A Study of the Internal Structure of Neurons using Punctual Graph Topological Space

In the field of neuroscience, neurons are considered the basic unit of the nervous system, playing a crucial role in processing information and executing neural functions. However, understanding the structure and function of neurons is a significant challenge due to their complex structural and functional properties. In this context, punctual graph topological space is a powerful mathematical tool for understanding the structure and function of neurons. By representing a neuron as a graph and then generated a punctual graph topological space from a graph, we can gain a better understanding of the relationship between the structure and function of neurons. We will provide a detailed explanation of the application of punctual graph topological space to the neurons.



Anatomical types of neurons

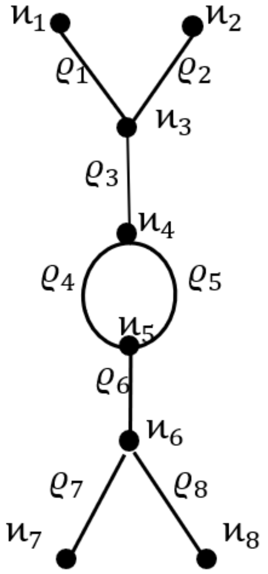


Fig.6 (Graph of Bipolar Neuron)

By using the graphical representation of bipolar neuron, we have
$I(\varrho_1) = \{\varkappa_1, \varkappa_3\}$, $I(\varrho_2) = \{\varkappa_2, \varkappa_3\}$, $I(\varrho_3) = \{\varkappa_3, \varkappa_4\}$, $I(\varrho_4) = \{\varkappa_4, \varkappa_5\}$, $I(\varrho_5) = \{\varkappa_4, \varkappa_5\}$, $I(\varrho_6) = \{\varkappa_5, \varkappa_6\}$, $I(\varrho_7) = \{\varkappa_6, \varkappa_7\}$, $I(\varrho_8) = \{\varkappa_6, \varkappa_8\}$

Then the sub basis $S_{I(\varrho)}$ is as follows:
$S_{I(\varrho)}$= $\{\emptyset,\{\varkappa_1,\varkappa_3\},\{\varkappa_2,\varkappa_3\},\{\varkappa_3,\varkappa_4\},\{\varkappa_4,\varkappa_5\},\{\varkappa_5,\varkappa_6\},\{\varkappa_6,\varkappa_8\},\{\varkappa_6,\varkappa_7\},\{\varkappa_3\},\{\varkappa_4\},\{\varkappa_5\},\{\varkappa_6\}\}$
Therefore, the topology on the space Ǵ can be written as:
τ_g =$\{\emptyset, \{\varkappa_1,\varkappa_3\}, \{\varkappa_2,\varkappa_3\}, \{\varkappa_3,\varkappa_4\}, \{\varkappa_4,\varkappa_5\}, \{\varkappa_5,\varkappa_6\}, \{\varkappa_6,\varkappa_8\}, \{\varkappa_6,\varkappa_7\}, \{\varkappa_3\}, \{\varkappa_4\}, \{\varkappa_5\}, \{\varkappa_6\},$
$\{\varkappa_3,\varkappa_5\}, \{\varkappa_3,\varkappa_6\}, \{\varkappa_4,\varkappa_6\}, \{\varkappa_1,\varkappa_3,\varkappa_4\}, \{\varkappa_1,\varkappa_2,\varkappa_3\}, \{\{\varkappa_1,\varkappa_3,\varkappa_5\}, \{\varkappa_1,\varkappa_3,\varkappa_6\}, \{\varkappa_2,\varkappa_3,\varkappa_4\},$
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$\{\varkappa_6,\varkappa_7,\varkappa_8\}, \{\varkappa_3,\varkappa_5,\varkappa_6\}, \{\varkappa_3,\varkappa_6,\varkappa_7\}, \{\varkappa_3,\varkappa_6,\varkappa_8\}, \{\varkappa_4,\varkappa_6,\varkappa_7\}, \{\varkappa_4,\varkappa_6,\varkappa_8\}, \{\varkappa_1,\varkappa_2,\varkappa_3,\varkappa_4\},$
$\{\varkappa_1,\varkappa_2,\varkappa_3,\varkappa_5\}, \{\varkappa_1,\varkappa_2,\varkappa_3,\varkappa_6\}, \{\varkappa_1,\varkappa_3,\varkappa_4,\varkappa_5\}, \{\varkappa_1,\varkappa_3,\varkappa_4,\varkappa_6\}, \{\varkappa_1,\varkappa_3,\varkappa_5,\varkappa_6\}, \{\varkappa_1,\varkappa_3,\varkappa_6,\varkappa_7\},$
$\{\varkappa_2,\varkappa_3,\varkappa_4,\varkappa_5\}, \{\varkappa_2,\varkappa_3,\varkappa_4,\varkappa_6\}, \{\varkappa_2,\varkappa_3,\varkappa_5,\varkappa_6\}, \{\varkappa_2,\varkappa_3,\varkappa_6,\varkappa_7\}, \{\varkappa_2,\varkappa_3,\varkappa_6,\varkappa_8\}, \{\varkappa_3,\varkappa_4,\varkappa_5,\varkappa_6\},$
$\{\varkappa_3,\varkappa_4,\varkappa_6,\varkappa_7\}, \{\varkappa_3,\varkappa_4,\varkappa_6,\varkappa_8\}, \{\varkappa_4,\varkappa_5,\varkappa_6,\varkappa_7\}, \{\varkappa_4,\varkappa_5,\varkappa_6,\varkappa_8\}, \{\varkappa_5,\varkappa_6,\varkappa_7,\varkappa_8\}, \{\varkappa_3,\varkappa_5,\varkappa_6,\varkappa_7\},$

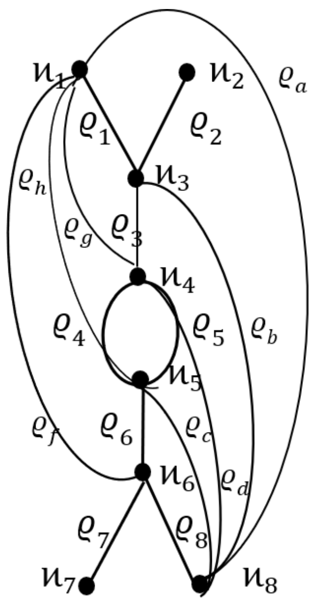
$\{u_3,u_5,u_6,u_8\},\{u_3,u_6,u_7,u_8\},\{u_4,u_6,u_7,u_8\},\{u_1,u_3,u_4,u_5,u_6\},\{u_1,u_3,u_4,u_6,u_7\},$
$\{u_1,u_3,u_4,u_6,u_8\},\{u_1,u_3,u_5,u_6,u_7\},\{n_1,u_3,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_2,u_3,u_4,u_6\},$
$\{u_1,u_2,u_3,u_5,u_6\},\{u_1,u_2,u_3,u_6,u_7\},\{n_1,u_2,u_3,u_6,u_8\}\{u_1,u_3,u_6,u_7,u_8\},\{u_2,u_3,u_4,u_5,u_6\},$
$\{u_2,u_3,u_4,u_6,u_7\},\{u_2,u_3,u_4,u_6,u_8\},\{u_2,u_3,u_5,u_6,u_7\},\{u_2,u_3,u_5,u_6,u_8\},\{u_2,u_3,u_6,u_7,u_8\},$
$\{u_3,u_4,u_5,u_6,u_7\},\{u_3,u_4,u_5,u_6,u_8\},\{u_3,u_4,u_6,u_7,u_8\},\{u_4,u_5,u_6,u_7,u_8\},\{u_3,u_5,u_6,u_7,u_8\},$
$\{u_1,u_3,u_4,u_5,u_6,u_7\},\{u_1,u_3,u_4,u_5,u_6,u_8\},\{u_1,u_3,u_4,u_6,u_7,u_8\}, \{u_1,u_3,u_5,u_6,u_7,u_8\},$
$\{u_2,u_3,u_4,u_5,u_6,u_7\},\{u_2,u_3,u_4,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_4,u_5,u_6\},\{u_1,u_2,u_3,u_4,u_6,u_7\},$
$\{u_1,u_2,u_3,u_4,u_6,u_8\},\{u_1,u_2,u_3,u_5,u_6,u_7\},\{u_1,u_2,u_3,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_6,u_7,u_8\},$
$\{u_2,u_3,u_4,u_6,u_7,u_8\}, \{u_2,u_3,u_5,u_6,u_7,u_8\}, \{u_3,u_4,u_5,u_6,u_7,u_8\},$
$\{u_1,u_3,u_4,u_5,u_6,u_7,u_8\},\{u_1,u_3,u_6,u_8\},,\{u_2,u_3,u_4,u_5,u_6,u_7,u_8\},\acute{G}\}$

Now, we find τ_g^i={family of i −open sets}

Hence

$\tau_g^i=\{\emptyset,\{u_1\},\{u_8\},\{u_1,u_4\},\{u_1,u_5\},\{u_1,u_6\},\{u_1,u_8\},\{u_3,u_8\},\{u_4,u_8\},\{u_5,u_8\},\{u_1,u_3,u_8\},$
$\{u_1,u_4,u_5\}, \{u_1,u_4,u_6\}, \{u_1,u_4,u_8\}, \{u_1,u_5,u_6\}, \{u_1,u_5,u_8\}, \{u_1,u_6,u_8\}, \{u_3,u_4,u_8\},$
$\{u_3,u_5,u_8\}, \{u_3,u_6,u_8\}, \{u_4,u_5,u_8\}, \{u_4,u_6,u_8\}, \{u_1,u_3,u_4,u_8\}, \{u_1,u_4,u_5,u_6\},$
$\{u_1,u_4,u_5,u_8\},\{u_1,u_4,u_6,u_8\},\{u_1,u_5,u_6,u_8\},\{u_1,u_3,u_4,u_5,u_8\},\{u_1,u_3\},\{u_2,u_3\},\{u_3,u_4\},$
$\{u_4,u_5\}, \{u_5,u_6\}, \{u_6,u_8\}, \{u_6,u_7\}, \{u_3\}, \{u_4\}, \{u_5\}, \{u_6\}, \{u_3,u_5\}, \{u_3,u_6\}, \{u_4,u_6\},$
$\{u_1,u_3,u_4\},\{u_1,u_2,u_3\}, \{\{u_1,u_3,u_5\}, \{u_1,u_3,u_6\}, \{u_2,u_3,u_4\},$
$\{u_2,u_3,u_5\},\{u_2,u_3,u_6\},\{u_3,u_4,u_5\},\{u_3,u_4,u_6\},\{u_4,u_5,u_6\},\{u_5,u_6,u_7\},\{u_5,u_6,u_8\},$
$\{u_6,u_7,u_8\},\{u_3,u_5,u_6\},\{u_3,u_6,u_7\},\{u_3,u_6,u_8\},\{u_4,u_6,u_7\},\{u_4,u_6,u_8\},\{u_1,u_2,u_3,u_4\},$
$\{u_1,u_2,u_3,u_5\},\{u_1,u_2,u_3,u_6\},\{u_1,u_3,u_4,u_5\},\{u_1,u_3,u_4,u_6\},\{u_1,u_3,u_5,u_6\},\{u_1,u_3,u_6,u_7\},$
$\{u_2,u_3,u_4,u_5\},\{u_2,u_3,u_4,u_6\},\{u_2,u_3,u_5,u_6\},\{u_2,u_3,u_6,u_7\},\{u_2,u_3,u_6,u_8\},\{u_3,u_4,u_5,u_6\},$
$\{u_3,u_4,u_6,u_7\},\{u_3,u_4,u_6,u_8\},\{u_4,u_5,u_6,u_7\},\{u_4,u_5,u_6,u_8\},\{u_5,u_6,u_7,u_8\},\{u_3,u_5,u_6,u_7\},$
$\{u_3,u_5,u_6,u_8\},\{u_3,u_6,u_7,u_8\},\{u_4,u_6,u_7,u_8\},\{u_1,u_3,u_4,u_5,u_6\},\{u_1,u_3,u_4,u_6,u_7\},$
$\{u_1,u_3,u_4,u_6,u_8\},\{u_1,u_3,u_5,u_6,u_7\},\{n_1,u_3,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_4,u_5\},\{u_1,u_2,u_3,u_4,u_6\},$
$\{u_1,u_2,u_3,u_5,u_6\},\{u_1,u_2,u_3,u_6,u_7\},\{n_1,u_2,u_3,u_6,u_8\}\{u_1,u_3,u_6,u_7,u_8\},\{u_2,u_3,u_4,u_5,u_6\},$
$\{u_2,u_3,u_4,u_6,u_7\},\{u_2,u_3,u_4,u_6,u_8\},\{u_2,u_3,u_5,u_6,u_7\},\{u_2,u_3,u_5,u_6,u_8\},\{u_2,u_3,u_6,u_7,u_8\},$
$\{u_3,u_4,u_5,u_6,u_7\},\{u_3,u_4,u_5,u_6,u_8\},\{u_3,u_4,u_6,u_7,u_8\},\{u_4,u_5,u_6,u_7,u_8\},\{u_3,u_5,u_6,u_7,u_8\},$
$\{u_1,u_3,u_4,u_5,u_6,u_7\},\{u_1,u_3,u_4,u_5,u_6,u_8\},\{u_1,u_3,u_4,u_6,u_7,u_8\}, \{u_1,u_3,u_5,u_6,u_7,u_8\},$
$\{u_2,u_3,u_4,u_5,u_6,u_7\},\{u_2,u_3,u_4,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_4,u_5,u_6\},\{u_1,u_2,u_3,u_4,u_6,u_7\},$
$\{u_1,u_2,u_3,u_4,u_6,u_8\},\{u_1,u_2,u_3,u_5,u_6,u_7\},\{u_1,u_2,u_3,u_5,u_6,u_8\},\{u_1,u_2,u_3,u_6,u_7,u_8\},$
$\{u_2,u_3,u_4,u_6,u_7,u_8\}, \{u_2,u_3,u_5,u_6,u_7,u_8\}, \{u_3,u_4,u_5,u_6,u_7,u_8\},$
$\{u_1,u_3,u_4,u_5,u_6,u_7,u_8\},\{u_1,u_3,u_6,u_8\},,\{u_2,u_3,u_4,u_5,u_6,u_7,u_8\},\acute{G}\}$

From the above, we deduce that the punctual graph topological space of the neuron is an expansible topological space, which allows us to expansible the original shape of the neuron. Consequently, its shape will slightly change when redrawn from the expansible punctual graph topological space, where new edges will be added and denoted by the following symbols $\varrho_a,\varrho_b,\varrho_a,\varrho_b,\varrho_a,\varrho_b,\varrho_a,\varrho_b$, and its shape will become as follows:



This work may provide insights into how the structure of neurons can change under different physiological and pathological conditions.

4-NON-EPANSIBLE PUNCTUAL GRAPH TOPOLOGICAL SPACE

In this context, we will outline our method for reviewing non-expansible punctual graph topological spaces, particularly the topological space of the transition graph for neurons and residential regions.

Definition 4.1 A punctual graph topological space $Ɠ = (N, Ę)$ is said to be non-expansible punctual graph topological space if $\tau_g = \tau_g^i$.

Example 4.2 Look at the graph $Ɠ = (N, Ę)$, as shown in the following

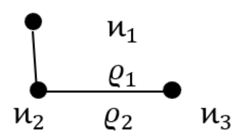


Fig.7

Let $\acute{Ɠ} = \{ɲ_1, ɲ_2, ɲ_3\}$, hence

$I(\varrho_1) = \{ɲ_1, ɲ_2\}$, $I(\varrho_2) = \{ɲ_2, ɲ_3\}$

Then the sub basis $S_{I(\varrho)}$ is as follows:

$S_{I(\varrho)}$= {∅,$\{ɲ_1, ɲ_2\}$,$\{ɲ_2, ɲ_3\}$,$\{ɲ_2\}$}

Therefore, the topology on the space $\acute{Ɠ}$ can be written as:

τ_g={∅,$\{ɲ_1, ɲ_2\}$,$\{ɲ_2, ɲ_3\}$,$\{ɲ_2\}$,$\acute{Ɠ}$}

Now, we find τ_g^i

τ_g^i={∅,{$ӥ_1, ӥ_2$},{$ӥ_2, ӥ_3$},{$ӥ_2$}, Ǵ}

Thus, we have $\tau_g = \tau_g^i$. This means that (Ǵ, τ_g) is a non-expansible punctual graph topological space.

4.3 Transition Graph of residential region

We will take the transition graph of the residential region, considering that nodes will be denoted by the symbol ϱ, and edges will be denoted by the symbol a, as shown in the following:

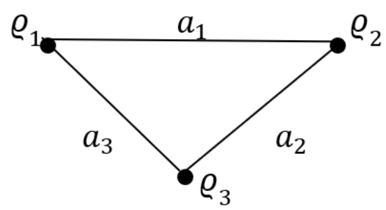


Fig.8

Let Ǵ = {$\varrho_1, \varrho_2, \varrho_3$}, hence

$I(a_1) = \{\varrho_1, \varrho_3\}$, $I(a_2) = \{\varrho_1, \varrho_2\}$, $I(a_3) = \{\varrho_2, \varrho_3\}$

Then the sub basis $S_{I(a)}$ is as follows:

$S_{I(a)}$= {∅,{ϱ_1, ϱ_3},{ϱ_1, ϱ_2},{ϱ_2, ϱ_3},{ϱ_1},{ϱ_2},{ϱ_3}}

Therefore, the topology on the space Ǵ can be written as:

τ_g={∅,{ϱ_1, ϱ_3},{ϱ_1, ϱ_2},{ϱ_2, ϱ_3},{ϱ_1},{ϱ_2},{ϱ_3},Ǵ}

We note that the topological space of the transition graph for residential region cannot be expansible because we will have $\tau_g = \tau_g^i$.

4.4 Transition Graph of Neuron

Our method for explaining the transition graph of the bipolar neuron, as shown in the following:

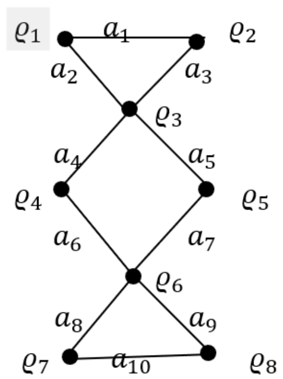


Fig.9

Let $\acute{G} = \{\varrho_1, \varrho_2, \varrho_3, \varrho_4, \varrho_5, \varrho_6, \varrho_7, \varrho_8\}$, hence

$I(a_1) = \{\varrho_1, \varrho_2\}$, $I(a_2) = \{\varrho_1, \varrho_3\}$, $I(a_3) = \{\varrho_2, \varrho_3\}$, $I(a_4) = \{\varrho_3, \varrho_4\}$, $I(a_5) = \{\varrho_3, \varrho_5\}$, $I(a_6) = \{\varrho_4, \varrho_6\}$

, $I(a_7) = \{\varrho_5, \varrho_6\}$, $I(a_8) = \{\varrho_6, \varrho_7\}$, $I(a_9) = \{\varrho_6, \varrho_8\}$, $I(a_{10}) = \{\varrho_7, \varrho_8\}$

Then the sub basis $S_{I(a)}$ is as follows:

$S_{I(a)} = \{\emptyset, \{\varrho_1, \varrho_2\}, \{\varrho_1, \varrho_3\}, \{\varrho_2, \varrho_3\}, \{\varrho_3, \varrho_4\}, \{\varrho_3, \varrho_5\}, \{\varrho_4, \varrho_6\}, \{\varrho_5, \varrho_6\}, \{\varrho_6, \varrho_7\}, \{\varrho_6, \varrho_8\}, \{\varrho_7, \varrho_8\}, \{\varrho_1\}, \{\varrho_2\}, \{\varrho_3\}, \{\varrho_4\}, \{\varrho_5\}, \{\varrho_6\}, \{\varrho_7\}, \{\varrho_8\}\}$

Therefore, the topology on the space $\acute{G}$ can be written as:

$\tau_g = \{\emptyset, \{\varrho_1, \varrho_2\}, \{\varrho_1, \varrho_3\}, \{\varrho_2, \varrho_3\}, \{\varrho_3, \varrho_4\}, \{\varrho_3, \varrho_5\}, \{\varrho_4, \varrho_6\}, \{\varrho_5, \varrho_6\}, \{\varrho_6, \varrho_7\}, \{\varrho_6, \varrho_8\}, \{\varrho_7, \varrho_8\}, \{\varrho_1\}, \{\varrho_2\}, \{\varrho_3\}, \{\varrho_4\}, \{\varrho_5\}, \{\varrho_6\}, \{\varrho_7\}, \{\varrho_8\}, \{\varrho_1, \varrho_4\}, \{\varrho_1, \varrho_5\}, \{\varrho_1, \varrho_6\}, \{\varrho_1, \varrho_7\}, \{\varrho_1, \varrho_8\}, \{\varrho_2, \varrho_4\}, \{\varrho_2, \varrho_5\}, \{\varrho_2, \varrho_6\}, \{\varrho_2, \varrho_7\}, \{\varrho_2, \varrho_8\}, \{\varrho_3, \varrho_6\}, \{\varrho_3, \varrho_7\}, \{\varrho_3, \varrho_8\}, \{\varrho_4, \varrho_5\}, \{\varrho_4, \varrho_7\}, \{\varrho_4, \varrho_8\}, \{\varrho_5, \varrho_7\}, \{\varrho_5, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_3\}, \{\varrho_1, \varrho_2, \varrho_4\}, \{\varrho_1, \varrho_2, \varrho_5\}, \{\varrho_1, \varrho_2, \varrho_6\}, \{\varrho_1, \varrho_2, \varrho_7\}, \{\varrho_1, \varrho_2, \varrho_8\}, \{\varrho_1, \varrho_3, \varrho_4\}, \{\varrho_1, \varrho_3, \varrho_5\}, \{\varrho_1, \varrho_3, \varrho_6\}, \{\varrho_1, \varrho_3, \varrho_7\}, \{\varrho_1, \varrho_3, \varrho_8\}, \{\varrho_1, \varrho_4, \varrho_5\}, \{\varrho_1, \varrho_4, \varrho_6\}, \{\varrho_1, \varrho_4, \varrho_7\}, \{\varrho_1, \varrho_4, \varrho_8\}, \{\varrho_1, \varrho_5, \varrho_6\}, \{\varrho_1, \varrho_5, \varrho_7\}, \{\varrho_1, \varrho_5, \varrho_8\}, \{\varrho_1, \varrho_6, \varrho_7\}, \{\varrho_1, \varrho_6, \varrho_8\}, \{\varrho_1, \varrho_7, \varrho_8\}, \{\varrho_2, \varrho_3, \varrho_4\}, \{\varrho_2, \varrho_3, \varrho_5\}, \{\varrho_2, \varrho_3, \varrho_6\}, \{\varrho_2, \varrho_3, \varrho_7\}, \{\varrho_2, \varrho_3, \varrho_8\}, \{\varrho_2, \varrho_4, \varrho_5\}, \{\varrho_2, \varrho_4, \varrho_6\}, \{\varrho_2, \varrho_4, \varrho_7\}, \{\varrho_2, \varrho_4, \varrho_8\}, \{\varrho_2, \varrho_5, \varrho_6\}, \{\varrho_2, \varrho_5, \varrho_7\}, \{\varrho_2, \varrho_5, \varrho_8\}, \{\varrho_2, \varrho_6, \varrho_7\}, \{\varrho_2, \varrho_6, \varrho_8\}, \{\varrho_2, \varrho_7, \varrho_8\}, \{\varrho_3, \varrho_4, \varrho_5\}, \{\varrho_3, \varrho_4, \varrho_6\}, \{\varrho_3, \varrho_4, \varrho_7\}, \{\varrho_3, \varrho_4, \varrho_8\}, \{\varrho_3, \varrho_5, \varrho_6\}, \{\varrho_3, \varrho_5, \varrho_7\}, \{\varrho_3, \varrho_5, \varrho_8\}, \{\varrho_3, \varrho_6, \varrho_7\}, \{\varrho_3, \varrho_6, \varrho_8\}, \{\varrho_3, \varrho_7, \varrho_8\}, \{\varrho_4, \varrho_5, \varrho_6\}, \{\varrho_4, \varrho_5, \varrho_7\}, \{\varrho_4, \varrho_5, \varrho_8\}, \{\varrho_4, \varrho_6, \varrho_7\}, \{\varrho_4, \varrho_6, \varrho_8\}, \{\varrho_4, \varrho_7, \varrho_8\}, \{\varrho_5, \varrho_6, \varrho_7\}, \{\varrho_5, \varrho_6, \varrho_8\}, \{\varrho_5, \varrho_7, \varrho_8\}, \{\varrho_6, \varrho_7, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_3, \varrho_4\}, \{\varrho_1, \varrho_2, \varrho_3, \varrho_5\}, \{\varrho_1, \varrho_2, \varrho_3, \varrho_6\}, \{\varrho_1, \varrho_2, \varrho_3, \varrho_7\}, \{\varrho_1, \varrho_2, \varrho_3, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_4, \varrho_5\}, \{\varrho_1, \varrho_2, \varrho_4, \varrho_6\}, \{\varrho_1, \varrho_2, \varrho_4, \varrho_7\}, \{\varrho_1, \varrho_2, \varrho_4, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_5, \varrho_6\}, \{\varrho_1, \varrho_2, \varrho_5, \varrho_7\}, \{\varrho_1, \varrho_2, \varrho_5, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_6, \varrho_7\}, \{\varrho_1, \varrho_2, \varrho_6, \varrho_8\}, \{\varrho_1, \varrho_2, \varrho_7, \varrho_8\}, \{\varrho_1, \varrho_3, \varrho_4, \varrho_5\}, \{\varrho_1, \varrho_3, \varrho_4, \varrho_6\}, \{\varrho_1, \varrho_3, \varrho_4, \varrho_7\}, \{\varrho_1, \varrho_3, \varrho_4, \varrho_8\}, \{\varrho_1, \varrho_3, \varrho_5, \varrho_6\}, \{\varrho_1, \varrho_3, \varrho_5, \varrho_7\}, \{\varrho_1, \varrho_3, \varrho_5, \varrho_8\}, \{\varrho_1, \varrho_3, \varrho_6, \varrho_7\}, \{\varrho_1, \varrho_3, \varrho_6, \varrho_8\}, \{\varrho_1, \varrho_3, \varrho_7, \varrho_8\}, \{\varrho_1, \varrho_4, \varrho_5, \varrho_6\}, \{\varrho_1, \varrho_4, \varrho_5, \varrho_7\}, \{\varrho_1, \varrho_4, \varrho_5, \varrho_8\}, \{\varrho_1, \varrho_4, \varrho_6, \varrho_7\}, \{\varrho_1, \varrho_4, \varrho_6, \varrho_8\}, \{\varrho_1, \varrho_4, \varrho_7, \varrho_8\}, \{\varrho_1, \varrho_5, \varrho_6, \varrho_7\}, \{\varrho_1, \varrho_5, \varrho_6, \varrho_8\}, \{\varrho_1, \varrho_5, \varrho_7, \varrho_8\}, \{\varrho_1, \varrho_6, \varrho_7, \varrho_8\}, \{\varrho_2, \varrho_3, \varrho_4, \varrho_5\}, \{\varrho_2, \varrho_3, \varrho_4, \varrho_6\}, \{\varrho_2, \varrho_3, \varrho_4, \varrho_7\}, \{\varrho_2, \varrho_3, \varrho_4, \varrho_8\}, \{\varrho_2, \varrho_3, \varrho_5, \varrho_6\}, \{\varrho_2, \varrho_3, \varrho_5, \varrho_7\}, \{\varrho_2, \varrho_3, \varrho_5, \varrho_8\}, \{\varrho_2, \varrho_3, \varrho_6, \varrho_7\}, \{\varrho_2, \varrho_3, \varrho_6, \varrho_8\}, \{\varrho_2, \varrho_3, \varrho_7, \varrho_8\}, \{\varrho_2, \varrho_4, \varrho_5, \varrho_6\}, \{\varrho_2, \varrho_4, \varrho_5, \varrho_7\}, \{\varrho_2, \varrho_4, \varrho_5, \varrho_8\}, \{\varrho_2, \varrho_4, \varrho_6, \varrho_7\}, \{\varrho_2, \varrho_4, \varrho_6, \varrho_8\}, \{\varrho_2, \varrho_4, \varrho_7, \varrho_8\}, \{\varrho_2, \varrho_5, \varrho_6, \varrho_7\}, \{\varrho_2, \varrho_5, \varrho_6, \varrho_8\}, \{\varrho_2, \varrho_5, \varrho_7, \varrho_8\}, \{\varrho_2, \varrho_6, \varrho_7, \varrho_8\}, \{\varrho_3, \varrho_4, \varrho_5, \varrho_6\}, \{\varrho_3, \varrho_4, \varrho_5, \varrho_7\}, \{\varrho_3, \varrho_4, \varrho_5, \varrho_8\}, \{\varrho_3, \varrho_4, \varrho_6, \varrho_7\},$

$\{\varrho_3,\varrho_4,\varrho_6,\varrho_8\}$, $\{\varrho_3,\varrho_4,\varrho_7,\varrho_8\}$, $\{\varrho_3,\varrho_5,\varrho_6,\varrho_7\}$, $\{\varrho_3,\varrho_5,\varrho_6,\varrho_8\}$, $\{\varrho_3,\varrho_5,\varrho_7,\varrho_8\}$, $\{\varrho_3,\varrho_6,\varrho_7,\varrho_8\}$, $\{\varrho_4,\varrho_5,\varrho_6,\varrho_7\}$, $\{\varrho_4,\varrho_5,\varrho_6,\varrho_8\}$, $\{\varrho_4,\varrho_5,\varrho_7,\varrho_8\}$, $\{\varrho_4,\varrho_6,\varrho_7,\varrho_8\}$, $\{\varrho_5,\varrho_6,\varrho_7,\varrho_8\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_4,\varrho_5\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_4,\varrho_6\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_4,\varrho_7\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_4,\varrho_8\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_5,\varrho_6\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_5,\varrho_7\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_5,\varrho_8\}$, $\{\varrho_1,\varrho_2,\varrho_3,\varrho_6,\varrho_7\}$, 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$\because \tau_g = P(\acute{G})$, hence $\tau_g = \tau_g^i$, which implies that the transition graph for neurons cannot be generalized.

Remark. The punctual graph topological space of the transition graphs is

1. $T_0 -$ space because Assume that there exist two distinct points x and y in the space Ǵ, such that $x \neq y$. Then, there exists an open set $\{x\}$ that

contains x but not y, or there exists an open set $\{y\}$ that contains y but not x.

2. $T_1 -$ space because Assume that there exist two distinct points x and y in the space Ǵ, such that $x \neq y$. Then, there exist two open sets $\{x\}$ and $\{y\}$ such that the set $\{x\}$ contains x but not y, and the set $\{y\}$ contains y but not x.

Conclusions

In this work, we showed that the punctual graph topological space can be an expansible and non-expansible topological space. We also found that the punctual graph topological space helps in improving urban planning and understanding the structure of neurons. Additionally, we demonstrated that the expansible topological space is a $T_0 -$ space and $T_1 -$ space.

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