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New Linear code From Projective Plane Over $GF(13)$

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A B S T R A C T

The aim of this paper is to create projective linear codes from parameters (n, M, d), based on the Galois field $GF(13)$ and its Application to Error – Correcting code.

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Keywords:

projective plane, coding theory, sphere packing, e-error correcting.

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الشفرات الخطية الجديدة الناشئة من المستوي الاسقاطي على حقل كالوا من الرتبة 13

فارس طلال عبد القادر احمد[(1)]، ندى ياسين قاسم يحيى[(2)]

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الخلاصة:

الهدف من هذا البحث هو إنشاء شفرات خطية إسقاطية من المعلمات (n, M, d), بناءً على حقل كالوا $\mathrm{GF}(13)$ وتطبيقه على شفرة تصحيح الخطأ.

الكلمات المفتاحية: الفضاء الاسقاطي، نظرية الترميز، كرة متراصة، شفرة تصحيح الخطأ.

1. INTRODUCTION

In symbology theory, many scientists have been studied a planar Galois field projective to a finite field for example Hirschfeld [1],[2] Many researchers have studied the theories and definitions between projective geometry and coding theory. AL- Seraji [3] Two papers presented important results on the relationship between the projective plane of order 17 and the error correction code. AL-Zangana [4] Classification of some concepts and study of the tools of the notation theorem.

They presented the relationship the projective plane and command 19 - correcting the code and the error values. Also, Yahya and AL-Zangana studied linear codes [5],[6]. In this research, A q-ary (n , M , d) code is an error - correcting code in coding theory that code a message of length m using q symbols (i.e. elements of a finite field with q elements) in to a code word of length n. the code is designed to detect and correct errors that many occur during transmission of the code word overy a noisy channel the minimum distance d is the minimum number of errors that can be corrected by the code.

1.1.Definition [2]: A code is e -error correcting if it can correct e errors.

1.2.Theorem [2]: (sphere packing or Hamming bound)
$A\ q-ary(n, M, 2e+2)-code\ C$ satisfies
$M\{\binom{n}{0}+\binom{n}{1}(q-1)+\cdots+\binom{n}{e}(q-1)^{e}\}\leq q^{n}$

1.3.Corollary [2]: $A\ q-ary\ (n, M, d)$ code C is perfect if and only if equality holds in Theorem 1.2

1.4.Definition [2]: $A\ q-ary\ code\ C$ The subset of length is n of $(\mathcal{F}_q)^n$

1.5.Definition [7]: let $\mathcal{F}(x)=x^n-a_{n-1}x^{n-1}-\cdots-a_0$ be a monic polynomial of degree n over $\mathcal{F}_q$. It's companion matrix of $\mathcal{F}(x)$ is given by the $n\times n$ matrix.

$$c(f)=\begin{bmatrix}0&1&0&\cdots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&0&1\\a_0&a_1&a_2&\cdots&a_{n-2}&a_{n-1}\end{bmatrix}_{n\times n}$$

2. Some results on a finite projective plane of order 13

The polynomial of degree three $g(x)=x^3-x^2-2$ is primitive in $\mathcal{F}_{13}=\{0,1,2,\ldots,10,11,12\}$, since $g(0)=11, g(1)=11, g(2)=2, g(3)=3, g(4)=7,\ g(5)=7, g(6)=9, g(7)=6, g(8)=4, g(9)=6, g(10)=1, g(11)=12, g(12)=9$,

The points and lines of $\mathcal{PG}(2,13)$ are generated as in the following:

$$p_i=[1,0,0]C(g)^{i-1}=[1,0,0]\begin{pmatrix}0&1&0\\0&0&1\\2&0&1\end{pmatrix}^{i-1},i=1,\ldots,183$$

Table 1: The point of $\mathcal{P}\mathrm{G}(2,13)$ are:

i	p_i	i	p_i	i	p_i	i	p_i
1	(1, 0, 0)	52	(1, 11, 3)	102	(1, 3, 4)	152	(1, 6, 6)
2	(0, 1, 0)	53	(1, 11, 11)	103	(1, 5, 9)	153	(1, 12, 1)
3	(0, 0, 1)	54	(1, 3, 1)	104	(1, 8, 8)	154	(1, 7, 0)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
50	(1, 11, 10)	100	(1, 9, 6)	150	(1, 9, 10)	182	(1, 12, 0)
51	(1, 2, 3)	101	(1, 12, 11)	151	(1, 2, 12)	183	(0, 1, 12)

With selecting the points in $\mathcal{P}G(2,13)$ which are the third coordinate equal to zero, this means belong to $\text{Ł}_0 = v(z)$, that is $v(z) = tz = z$ for all t in $\mathcal{F}_{13}\backslash\{0\}$ and with $p_i = i$, we obtain

$\text{Ł}_1 = \{1, 2, 9, 25, 38, 42, 60, 108, 120, 129, 135, 140, 154, 182\}$, that is

$$\text{Ł}_i = \text{Ł}_1 C(g)^{i-1} = \text{Ł}_1 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 2 & 0 & 1 \end{pmatrix}^{i-1} \quad , i = 1, \dots, 183$$

Table 2: The lines of $\mathcal{P}G(2,13)$ are:

Ł_1	1	2	9	25	38	42	60	108	120	129	135	140	154	182
Ł_2	2	3	10	26	39	43	61	109	121	130	136	141	155	183
Ł_3	3	4	11	27	40	44	62	110	122	131	137	142	156	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
Ł_{181}	181	182	6	22	35	39	57	105	117	126	132	137	151	179
Ł_{182}	182	183	7	23	36	40	58	106	118	127	133	138	152	180
Ł_{183}	183	1	8	24	37	41	59	107	119	128	134	139	153	181

In the following theorem the parameters n, M and d are constructed.

2.1. **Theorem :** The projective plane of order thirteen is a code with a parameters $[n = 183, M \leq 13^4, d = 14]$.

Proof: The plane π_{13} has an incidence matrix $A = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & if \quad p_j \in \text{Ł}_i \\ 0 & if \quad p_j \notin \text{Ł}_i \end{cases}$$

Table 3

Construct a table of points in $\mathcal{PG}(2,13)$ so that we the point that lies on line we put one and for the point that does not lie on the line we put zero.

$Ł_i$ \ p_i	p_1	p_2	p_3	...	p_{181}	p_{182}	p_{183}
$Ł_1$	1	1	0	...	0	1	0
$Ł_2$	0	1	1	...	0	0	1
$Ł_3$	1	0	1	...	0	0	0
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$Ł_{181}$	0	0	0	...	1	1	0
$Ł_{182}$	0	0	0	...	0	1	1
$Ł_{183}$	1	0	0	...	1	0	1

Let

ψ=[0,0,0,…,0], β=[1,1,1,…,1], φ=[2,2,2,…,2], μ=[3,3,3,…,3], η=[4,4,4,…,4], α=[5,5,5,…,5], Ω=[6,6,6,…,6], σ=[7,7,7,…,7], Ӿ=[8,8,8,…,8], ℏ=[9,9,9,…,9], ∮=[10,10,10,…,10], λ=[11,11,11,…,11], ϖ=[12,12,12,…,12]

$i = 1, \cdots, 183$

Table 4

$ɱ_i = \boldsymbol{\beta} + Ł_i$, That is ,

$ɱ_1 = 1 + 0 = 1$ *or* $ɱ_1 = 1 + 1 = 2$

$ɱ_1$	2	2	1	...	1	2	1
$ɱ_2$	1	2	2	...	1	1	2
$ɱ_3$	2	1	2	...	1	1	1
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$ɱ_{181}$	1	1	1	...	2	2	1
$ɱ_{182}$	1	1	1	...	1	2	2
$ɱ_{183}$	2	1	1	...	2	1	2

Table 5

$ꞩ_i = \varphi + Ł_i$, **That is ,**

$ꞩ_1 = 2 + 0 = 2 \quad or \quad ꞩ_1 = 2 + 1 = 3$

$ꞩ_1$	3	3	2	...	2	3	2
$ꞩ_2$	2	3	3	...	2	2	3
$ꞩ_3$	3	2	3	...	2	2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$ꞩ_{181}$	2	2	2	...	3	3	2
$ꞩ_{182}$	2	2	2	...	2	3	3
$ꞩ_{183}$	3	2	2	...	3	2	3

Table 6

$ŧ_i = \mu + Ł_i$, **That is ,**

$ŧ_1 = 3 + 0 = 3 \quad or \quad ŧ_1 = 3 + 1 = 4$

$ŧ_1$	4	4	3	…	3	4	3
$ŧ_2$	3	4	4	...	3	3	4
$ŧ_3$	4	3	4	...	3	3	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$ŧ_{181}$	3	3	3	...	4	4	3
$ŧ_{182}$	3	3	3	...	3	4	4
$ŧ_{183}$	4	3	3	...	4	3	4

Table 7

$\mathfrak{u}_i = \eta + Ł_i$, **That is ,**

$\mathfrak{u}_1 = 4 + 0 = 4 \quad or \quad \mathfrak{u}_1 = 4 + 1 = 5$

$\mathfrak{u}_1$	5	5	4	...	4	5	4
$\mathfrak{u}_2$	4	5	5	...	4	4	5
$\mathfrak{u}_3$	5	4	5	...	4	4	4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathfrak{u}_{181}$	4	4	4	...	5	5	4
$\mathfrak{u}_{182}$	4	4	4	...	4	5	5
$\mathfrak{u}_{183}$	5	4	4	...	5	4	5

Table 8

$\boldsymbol{v_i = \alpha + Ł_i}$, **That is ,**

$\boldsymbol{v_1 = 5 + 0 = 5 \quad or \quad v_1 = 5 + 1 = 6}$

v_1	6	6	5	...	5	6	5
v_2	5	6	6	...	5	5	6
v_3	6	5	6	...	5	5	5
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
v_{181}	5	5	5	...	6	6	5
v_{182}	5	5	5	...	5	6	6
v_{183}	6	5	5	...	6	5	6

Table 9

$\boldsymbol{w_i = \Omega + Ł_i}$, **That is ,**

$\boldsymbol{w_1 = 6 + 0 = 6 \quad or \quad w_1 = 6 + 1 = 7}$

w_1	7	7	6	...	6	7	6
w_2	6	7	7	...	6	6	7
w_3	7	6	7	...	6	6	6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
w_{181}	6	6	6	...	7	7	6
w_{182}	6	6	6	...	6	7	7
w_{183}	7	6	6	...	7	6	7

Table 10

$\boldsymbol{x_i = \sigma + Ł_i}$, **That is ,**

$\boldsymbol{x_1 = 7 + 0 = 7 \quad or \quad x_1 = 7 + 1 = 8}$

x_1	8	8	7	...	7	8	7
x_2	7	8	8	...	7	7	8
x_3	8	7	8	...	7	7	7
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
x_{181}	7	7	7	...	8	8	7
x_{182}	7	7	7	...	7	8	8
x_{183}	8	7	7	...	8	7	8

Table 11

$\mathcal{y}_i = \mathfrak{X} + Ł_i$, **That is ,**

$\boldsymbol{y_1 = 8 + 0 = 8 \quad or \quad y_1 = 8 + 1 = 9}$

y_1	9	9	8	...	8	9	8
y_2	8	9	9	...	8	8	9
y_3	9	8	9	...	8	8	8
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
y_{181}	8	8	8	...	9	9	8
y_{182}	8	8	8	...	8	9	9
y_{183}	9	8	8	...	9	8	9

Table 12

$z_i = \hbar + Ł_i$, **That is ,**

$\boldsymbol{z_1 = 9 + 0 = 9 \quad or \quad z_1 = 9 + 1 = 10}$

z_1	10	10	9	...	9	10	9
z_2	9	10	10	...	9	9	10
z_3	10	9	10	...	9	9	9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
z_{181}	9	9	9	...	10	10	9
z_{182}	9	9	9	...	9	10	10
z_{183}	10	9	9	...	10	9	10

Table 13

$k_i = \not{f} + Ł_i$, **That is ,**

$\boldsymbol{k_1 = 10 + 0 = 10 \quad or \quad k_1 = 10 + 1 = 11}$

k_1	11	11	10	...	10	11	10
k_2	10	11	11	...	10	10	11
k_3	11	10	11	...	10	10	10
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
k_{181}	10	10	10	...	11	11	10
k_{182}	10	10	10	...	10	11	11
k_{183}	11	10	10	...	11	10	11

Table 14

$ƞ_i = \lambda + Ł_i$, That is ,

$ƞ_1 = 11 + 0 = 11 \quad or \quad ƞ_1 = 11 + 1 = 12$

$ƞ_1$	12	12	11	...	11	12	11
$ƞ_2$	11	12	12	...	11	11	12
$ƞ_3$	12	11	12	...	11	11	11
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$ƞ_{181}$	11	11	11	...	12	12	11
$ƞ_{182}$	11	11	11	...	11	12	12
$ƞ_{183}$	12	11	11	...	12	11	12

Table 15

$r_i = \varpi + Ł_i$, That is ,

$r_1 = 12 + 0 = 12 \quad or \quad r_1 = 12 + 1 = 0$

r_1	0	0	12	...	12	0	12
r_2	12	0	0	...	12	12	0
r_3	0	12	0	...	12	12	12
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
r_{181}	12	12	12	...	0	0	12
r_{182}	12	12	12	...	12	0	0
r_{183}	0	12	12	...	0	12	0

The remain vectors of code $C$are constructed combination of $\psi, \beta, \varphi, \mu, \eta, \alpha, \Omega, \sigma$, Ӿ, $\hbar$, ∮, λ, ϖ, $Ł_i$, $ꝳ_i$, $ꞩ_i$, $ƫ_i$, u_i, v_i, w_i, $ӽ_i$, y_i, z_i, k_i, $ƞ_i$ and r_i where $i = 1, \ldots, 183$,

Note that $d(Ł_i, Ł_j)$ = number of points on exactly one of $Ł_i$ or $Ł_j$. Then

Table 16

the minimum distance d of a non-trivial code C.

$d(\psi, Ł_i)$=14	$d(u_i, ƫ_i)$=183	$d(\psi, ꝳ_i)$=183	$d(\sigma, u_i)$=183	$d(\psi, y_i)$=183
$d(\beta, Ł_i)$=169	$d(v_i, ƫ_i)$=183	$d(\beta, ꝳ_i)$=14	d(Ӿ, u_i)=183	$d(\beta, y_i)$=183
$d(\varphi, Ł_i)$=183	$d(w_i, ƫ_i)$=183	$d(\varphi, ꝳ_i)$=169	$d(\hbar, u_i)$=183	$d(\varphi, y_i)$=183
$d(\mu, Ł_i)$=183	$d(ӽ_i, ƫ_i)$=183	$d(\mu, ꝳ_i)$=183	d(∮, u_i)=183	$d(\mu, y_i)$=183
$d(\eta, Ł_i)$=183	$d(y_i, ƫ_i)$=183	$d(\eta, ꝳ_i)$=183	$d(\lambda, u_i)$=183	$d(\eta, y_i)$=183

$d(\alpha,Ƚ_i)=183$	$d(z_i,t_i)=183$	$d(\alpha,ᵯ_i)=183$	$d(\varpi,u_i)=183$	$d(\alpha,y_i)=183$
$d(\Omega,Ƚ_i)=183$	$d(ķ_i,t_i)=183$	$d(\Omega,ᵯ_i)=183$	$d(\psi,v_i)=183$	$d(\Omega,y_i)=183$
$d(\sigma,Ƚ_i)=183$	$d(ŋ_i,t_i)=183$	$d(\sigma,ᵯ_i)=183$	$d(\beta,v_i)=183$	$d(\sigma,y_i)=183$
$d(Ӿ,Ƚ_i)=183$	$d(r_i,t_i)=183$	$d(Ӿ,ᵯ_i)=183$	$d(\varphi,v_i)=183$	$d(Ӿ,y_i)=14$
$d(\hbar,Ƚ_i)=183$	$d(v_i,u_i)=183$	$d(\hbar, ᵯ_i)=183$	$d(\mu,v_i)=183$	$d(\hbar,y_i)=169$
$d(ꞙ,Ƚ_i)=183$	$d(w_i,u_i)=183$	$d(ꞙ,ᵯ_i)=183$	$d(\eta,v_i)=183$	$d(ꞙ,y_i)=183$
$d(\lambda,Ƚ_i)=183$	$d(ꭗ_i,u_i)=183$	$d(\lambda,ᵯ_i)=183$	$d(\alpha,v_i)=14$	$d(\lambda,y_i)=183$
$d(\varpi,Ƚ_i)=183$	$d(y_i,u_i)=183$	$d(\varpi,ᵯ_i)=183$	$d(\Omega,v_i)=169$	$d(\varpi,y_i)=183$
$d(ᵯ_i,Ƚ_i)=183$	$d(z_i,u_i)=183$	$d(\psi,ȿ_i)=183$	$d(\sigma,v_i)=183$	$d(\psi,z_i)=183$
$d(ȿ_i,Ƚ_i)=183$	$d(ķ_i,u_i)=183$	$d(\beta,ȿ_i)=183$	$d(Ӿ,v_i)=183$	$d(\beta,z_i)=183$
$d(t_i,Ƚ_i)=183$	$d(ŋ_i,u_i)=183$	$d(\varphi,ȿ_i)=14$	$d(\hbar,v_i)=183$	$d(\varphi,z_i)=183$
$d(u_i,Ƚ_i)=183$	$d(r_i,u_i)=183$	$d(\mu,ȿ_i)=169$	$d(ꞙ,v_i)=183$	$d(\mu,z_i)=183$
$d(v_i,Ƚ_i)=183$	$d(w_i,v_i)=183$	$d(\eta,ȿ_i)=183$	$d(\lambda,v_i)=183$	$d(\eta,z_i)=183$
$d(w_i,Ƚ_i)=183$	$d(ꭗ_i,v_i)=183$	$d(\alpha,ȿ_i)=183$	$d(\varpi,v_i)=183$	$d(\alpha,z_i)=183$
$d(ꭗ_i,Ƚ_i)=183$	$d(y_i,v_i)=183$	$d(\Omega,ȿ_i)=183$	$d(\psi,w_i)=183$	$d(\Omega,z_i)=183$
$d(y_i,Ƚ_i)=183$	$d(z_i,v_i)=183$	$d(\sigma,ȿ_i)=183$	$d(\beta,w_i)=183$	$d(\sigma,z_i)=183$
$d(z_i,Ƚ_i)=183$	$d(ķ_i,v_i)=183$	$d(Ӿ,ȿ_i)=183$	$d(\varphi,w_i)=183$	$d(Ӿ,z_i)=183$
$d(ķ_i,Ƚ_i)=183$	$d(ŋ_i,v_i)=183$	$d(\hbar,ȿ_i)=183$	$d(\mu,w_i)=183$	$d(\hbar, z_i)=14$
$d(ŋ_i,Ƚ_i)=183$	$d(r_i,v_i)=183$	$d(ꞙ,ȿ_i)=183$	$d(\eta,w_i)=183$	$d(ꞙ,z_i)=169$
$d(r_i,Ƚ_i)=183$	$d(ꭗ_i,w_i)=183$	$d(\lambda,ȿ_i)=183$	$d(\alpha,w_i)=183$	$d(\lambda,z_i)=183$
$d(ȿ_i,ᵯ_i)=183$	$d(y_i,w_i)=183$	$d(\varpi,ȿ_i)=183$	$d(\Omega,w_i)=14$	$d(\varpi,z_i)=183$

$d(t_i,ᵯ_i)=183$	$d(z_i,w_i)=183$	$d(\psi,t_i)=183$	$d(\sigma,w_i)=169$	$d(\psi,k_i)=183$
$d(u_i,ᵯ_i)=183$	$d(k_i,w_i)=183$	$d(\beta,t_i)=183$	$d(Ӿ,w_i)=183$	$d(\beta,k_i)=183$
$d(v_i,ᵯ_i)=183$	$d(n_i,w_i)=183$	$d(\varphi,t_i)=183$	$d(\hbar,w_i)=183$	$d(\varphi,k_i)=183$
$d(w_i,ᵯ_i)=183$	$d(r_i,w_i)=183$	$d(\mu,t_i)=14$	$d(ꞙ,w_i)=183$	$d(\mu,k_i)=183$
$d(ꭓ_i,ᵯ_i)=183$	$d(y_i,ꭓ_i)=183$	$d(\eta,t_i)=169$	$d(\lambda,w_i)=183$	$d(\eta,k_i)=183$
$d(y_i,ᵯ_i)=183$	$d(z_i,ꭓ_i)=183$	$d(\alpha,t_i)=183$	$d(\varpi,w_i)=183$	$d(\alpha,k_i)=183$
$d(z_i,ᵯ_i)=183$	$d(k_i,ꭓ_i)=183$	$d(\Omega,t_i)=183$	$d(\psi,ꭓ_i)=183$	$d(\Omega,k_i)=183$
$d(k_i,ᵯ_i)=183$	$d(n_i,ꭓ_i)=183$	$d(\sigma,t_i)=183$	$d(\beta,ꭓ_i)=183$	$d(\sigma,k_i)=183$
$d(n_i,ᵯ_i)=183$	$d(r_i,ꭓ_i)=183$	$d(Ӿ,t_i)=183$	$d(\varphi,ꭓ_i)=183$	$d(Ӿ,k_i)=183$
$d(r_i,ᵯ_i)=183$	$d(z_i,y_i)=183$	$d(\hbar,t_i)=183$	$d(\mu,ꭓ_i)=183$	$d(\hbar,k_i)=183$
$d(t_i,ꞩ_i)=183$	$d(k_i,y_i)=183$	$d(ꞙ,t_i)=183$	$d(\eta,ꭓ_i)=183$	$d(ꞙ,k_i)=14$
$d(u_i,ꞩ_i)=183$	$d(n_i,y_i)=183$	$d(\lambda,t_i)=183$	$d(\alpha,ꭓ_i)=183$	$d(\lambda,k_i)=169$
$d(v_i,ꞩ_i)=183$	$d(r_i,y_i)=183$	$d(\varpi,t_i)=183$	$d(\Omega,ꭓ_i)=183$	$d(\varpi,k_i)=183$
$d(w_i,ꞩ_i)=183$	$d(k_i,z_i)=183$	$d(\psi,u_i)=183$	$d(\sigma,ꭓ_i)=14$	$d(\psi,n_i)=183$
$d(ꭓ_i,ꞩ_i)=183$	$d(n_i,z_i)=183$	$d(\beta,u_i)=183$	$d(Ӿ,ꭓ_i)=169$	$d(\beta,n_i)=183$
$d(y_i,ꞩ_i)=183$	$d(r_i,z_i)=183$	$d(\varphi,u_i)=183$	$d(\hbar,ꭓ_i)=183$	$d(\varphi,n_i)=183$
$d(z_i,ꞩ_i)=183$	$d(n_i,k_i)=183$	$d(\mu,u_i)=183$	$d(ꞙ,ꭓ_i)=183$	$d(\mu,n_i)=183$
$d(k_i,ꞩ_i)=183$	$d(r_i,k_i)=183$	$d(\eta,u_i)=14$	$d(\lambda,ꭓ_i)=183$	$d(\eta,n_i)=183$
$d(n_i,ꞩ_i)=183$	$d(r_i,n_i)=183$	$d(\alpha,u_i)=169$	$d(\varpi,ꭓ_i)=183$	$d(\alpha,n_i)=183$
$d(r_i,ꞩ_i)=183$	$d(\sigma,n_i)=183$	$d(\Omega,u_i)=183$	$d(Ӿ,n_i)=183$	$d(\Omega,n_i)=183$
$d(\varphi,\mu)=183$	$d(\mu,\eta)=183$	$d(\eta,\alpha)=183$	$d(\alpha,\Omega)=183$	$d(\Omega,\sigma)=183$

$d(\varphi,\eta)$=183	$d(\mu,\alpha)$=183	$d(\eta,\Omega)$=183	$d(\alpha,\sigma)$=183	$d(\Omega,Ӿ)$=183
$d(\varphi,\alpha)$=183	$d(\mu,\Omega)$=183	$d(\eta,\sigma)$=183	$d(\alpha,Ӿ)$=183	$d(\Omega,\hbar)$=183
$d(\varphi,\Omega)$=183	$d(\mu,\sigma)$=183	$d(\eta,Ӿ)$=183	$d(\alpha,\hbar)$=183	$d(\Omega,ꞙ)$=183
$d(\varphi,\sigma)$=183	$d(\mu,Ӿ)$=183	$d(\eta,\hbar)$=183	$d(\alpha,ꞙ)$=183	$d(\Omega,\lambda)$=183
$d(\varphi,Ӿ)$=183	$d(\mu,\hbar)$=183	$d(\eta,ꞙ)$=183	$d(\alpha,\lambda)$=183	$d(\Omega,\varpi)$=183
$d(\varphi,\hbar)$=183	$d(\mu,ꞙ)$=183	$d(\eta,\lambda)$=183	$d(\alpha,\varpi)$=183	$d(\hbar,ɳ_i)$=183
$d(\varphi,ꞙ)$=183	$d(\mu,\lambda)$=183	$d(\eta,\varpi)$=183	$d(Ӿ,\hbar)$=183	$d(ꞙ,ɳ_i)$=183
$d(\varphi,\lambda)$=183	$d(\mu,\varpi)$=183	$d(\hbar,ꞙ)$=183	$d(Ӿ,ꞙ)$=183	$d(\lambda,ɳ_i)$=14
$d(\varphi,\varpi)$=183	$d(ꞙ,\lambda)$=183	$d(\hbar,\lambda)$=183	$d(Ӿ,\lambda)$=183	$d(\varpi,ɳ_i)$=169
$d(\lambda,\varpi)$=183	$d(ꞙ,\varpi)$=183	$d(\hbar,\varpi)$=183	$d(Ӿ,\varpi)$=183	$d(\psi,ɼ_i)$=169
$d(\beta,ɼ_i)$=183	$d(ꞙ,ɼ_i)$=183	$d(\sigma,Ӿ)$=183	$d(\psi,\sigma)$=183	$d(\beta,\Omega)$=183
$d(\varphi,ɼ_i)$=183	$d(\lambda,ɼ_i)$=183	$d(\sigma,\hbar)$=183	$d(\psi,Ӿ)$=183	$d(\beta,\sigma)$=183
$d(\mu,ɼ_i)$=183	$d(\varpi,ɼ_i)$=14	$d(\sigma,ꞙ)$=183	$d(\psi,\hbar)$=183	$d(\beta,Ӿ)$=183
$d(\eta,ɼ_i)$=183	$d(\psi,\beta)$=183	$d(\sigma,\lambda)$=183	$d(\psi,ꞙ)$=183	$d(\beta,\alpha)$=183
$d(\alpha,ɼ_i)$=183	$d(\psi,\varphi)$=183	$d(\sigma,\varpi)$=183	$d(\psi,\lambda)$=183	$d(\beta,\eta)$=183
$d(\Omega,ɼ_i)$=183	$d(\psi,\mu)$=183	$d(\beta,\hbar)$=183	$d(\psi,\varpi)$=183	$d(\beta,\varpi)$=183
$d(\sigma,ɼ_i)$=183	$d(\psi,\eta)$=183	$d(\beta,ꞙ)$=183	$d(\beta,\varphi)$=183	$d(\psi,\Omega)$=183
$d(Ӿ,ɼ_i)$=183	$d(\psi,\alpha)$=183	$d(\beta,\lambda)$=183	$d(\beta,\mu)$=183	$d(\hbar,ɼ_i)$=183

If we substitute the values of $n = 183$, $d = 14$, $\mathrm{e} = 6$, in inequality of theorem 1.2 , we get $M = q^{k+1} = 13^4$. Hence C is a $(183, 13^4, 14) - code$.

$$13^4\left\{\binom{183}{0}+\binom{183}{1}(13-1)+\binom{183}{2}(13-1)^2+\binom{183}{3}(13-1)^3+\binom{183}{4}(13-1)^4 +\binom{183}{5}(13-1)^5+\binom{183}{6}(13-1)^6\right\}$$
$$\leq 13^{183}$$

By Corollary(1.3) , therefore C is **perfect.**

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