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Applications of Coding Theory in Projective Plane of Order 17

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ABSTRACT

The goal of this research is to make connection between the projective plane of order 17 and coding theory.

تطبيقات نظرية الترميز في المستوى الاسقاطي من الرتبة 17

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الخلاصة:

الهدف من هذا العمل هو تقديم العلاقة بين المستوي الاسقاطي من الرتبة 17 ونظرية الترميز.

الكلمات المفتاحية: الفضاء الاسقاطي، نظرية الترميز، مصفوفة الوقوع، قيد هامنك

1. INTRODUCTION

In symbology theory, many scientists have been studied a planar Galois field projective to a finite field for example Hirschfeld [1],[2] Many researchers have studied the theories and definitions between projective geometry and coding theory. AL- Seraji [3] Two papers presented important results on the relationship between the projective level of command 17 and the error correction code. Classification of some concepts and study of the tools of the notation theorem [4].

They presented the relationship the projective level and command 19 - correcting the code and the error values . Also, Yahya and AL-Zangana studied linear codes [5],[6] . In this research ,A q-ary (n ,M ,d) code is an error - correcting code in coding theory that code a message of length m using q symbols (i.e. elements of a finite field with q elements) in to a code word of length n. the code is designed to detect and correct errors that many occur during transmission of the code word overy a noisy channel the minimum distance d is the minimum number of errors that can be corrected by the code.

1.1.Definition [2]: A code is e-error correcting if it can correct e errors.

1.2.Theorem[2]: (sphere packing or Hamming bound)

$A\ q-ary(n,M,2e+2)-code\ C$ satisfies

$\mathrm{M}\{\binom{n}{0}+\binom{n}{1}(q-1)+\cdots+\binom{n}{e}(q-1)^{e}\}\leq q^{n}$

1.3.Corollary [2]: $A\ q-ary\ (n,M,d)$ code C is perfect if and only if equality holds in Theorem 1.2

1.4.Definition [2]: $A\ q-ary\ code\ C$ The subset of length is n of $\left(F_q\right)^n$

1.5.Definition [7]: let $F(x)=x^n-b_{n-1}x^{n-1}-\cdots-b_0$ be a monic polynomial of degree n over F_q . It's companion matrix of $F(x)$ is given by the $n\times n$ matrix .

$$c(f)=\begin{bmatrix}0&1&0&\cdots&0&0\\0&0&1&\cdots&0&0\\\vdots&\vdots&\vdots&\vdots&\vdots&\vdots\\0&0&0&\cdots&0&1\\b_0&b_1&b_2&\cdots&b_{n-2}&b_{n-1}\end{bmatrix}_{n\times n}$$

2. NEW RESULTS

2.1. Cubic curve over a finite field of order 17

The polynomial of degree three $\mathcal{S}(x)=x^3-8x^2-1$ is primitive in $F_{17}=\{0,1,2,3,4,5,6,7,8,9,10,11,12,13,14,15,16\}$ since $\mathcal{S}(0)=16$, $\mathcal{S}(1)=9$, $\mathcal{S}(2)=9$, $\mathcal{S}(3)=5$, $\mathcal{S}(4)=3$, $\mathcal{S}(5)=9$, $\mathcal{S}(6)=12$, $\mathcal{S}(7)=1$, $\mathcal{S}(8)=16$, $\mathcal{S}(9)=12$, $\mathcal{S}(10)=12$, $\mathcal{S}(11)=5$, $\mathcal{S}(12)=14$, $\mathcal{S}(13)=11$, $\mathcal{S}(14)=2$, $\mathcal{S}(15)=10$, $\mathcal{S}(16)=7$, this means 1,2,3,4,5,6,7,8,9,10,11,12,13,14,15 are roots of $\mathcal{S}\ in\ F_{17}$.

The points and lines of $PG(2,17)$ are generated as in the following:

$$\mathcal{P}(i)=[1,0,0]C(\mathcal{S})^{i-1}=[1,0,0]\begin{pmatrix}0&1&0\\0&0&1\\1&0&8\end{pmatrix}^{i-1},i=1,2,\ldots\ldots307 .$$

Table 1: The point of $PG(2,17)$ are:

i	$\mathcal{P}_i$	i	$\mathcal{P}_i$	i	$\mathcal{P}_i$	i	$\mathcal{P}_i$	i	$\mathcal{P}_i$
1	(1,0, 0)	52	(1,1,4)	102	(1,3,4)	152	(1,8,0)	202	(1,8,4)
2	(0,1, 0)	53	(1,13,7)	103	(1,5,9)	153	(0,1,8)	203	(1,13,10)
3	(0,0,1)	54	(1,5,5)	104	(1,8,8)	154	(1,0,6)	204	(1,12,11)
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
50	(1,13,1)	100	(1,1,1)	150	(1,9,10)	200	(1,3,15)	306	(1,2,0)
51	(1,1,4)	101	(1,1,9)	151	(1,16,15)	201	(1,8,15)	307	(0,1,2)

After choosing the points in $PG(2,17)$ which are the third coordinate equal to zero, this means belong to $\mathfrak{L}_0 = \upsilon(z)$, that $\upsilon(z) = tz = z$ for all t in $F_{17}\backslash\{0\}$ and with $\mathcal{P}(i) = i$, we get

$\mathfrak{L}_1 = \{1,2,10,16,87,110,120,152,176,180,192,211,233,254,259,272,279,306\}$

$$\mathfrak{L}_i = \mathfrak{L}_1 C(\mathcal{S})^{i-1} = \mathfrak{L}_1 \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 8 \end{pmatrix}^{i-1} \quad , i = 1,2, \dots 307 .$$

Table 2: The lines of $PG(2,17)$ are:

$\mathfrak{L}_1$	1	2	10	16	87	110	120	152	176	180	192	211	233	254	259	272	279	306
$\mathfrak{L}_2$	2	3	11	17	88	111	121	153	177	181	193	212	234	255	260	273	280	307
$\mathfrak{L}_3$	3	4	12	18	89	112	122	154	178	182	194	213	235	256	261	274	281	1
$\mathfrak{L}_4$	4	5	13	19	90	113	123	155	179	183	195	214	236	257	262	275	282	2
⋮																		
⋮																		
$\mathfrak{L}_{304}$	304	305	6	12	83	106	116	148	172	176	188	207	229	250	255	268	275	302
$\mathfrak{L}_{305}$	305	306	7	13	84	107	117	149	173	177	189	208	230	251	256	269	276	303
$\mathfrak{L}_{306}$	306	307	8	14	85	108	118	150	174	178	190	209	231	252	257	270	277	304
$\mathfrak{L}_{307}$	307	1	9	15	86	109	119	151	175	179	191	210	232	253	258	271	278	305

In the following theorem the parameters n , M and d are constructed.

2.1.1. **Theorem :** The projective plane of order 17 is a code with a parameters $[n = 307, M = 17^4, d = 18]$

Proof: the plane π_{17} has an incidence matrix $A = (a_{ij})$, where

$$a_{ij} = \begin{cases} 1 & if \;\; P_j \in \ell_i \\ 0 & if \;\; P_j \notin \ell_i \end{cases}$$

Table 3: Construct a table of points in $PG(2,17)$ so that we the point that lies on line we put one and for the point that does not lie on the plane we put zero.

$\mathcal{P}_i$ / $\mathfrak{L}_i$	$\mathcal{P}_1$	$\mathcal{P}_2$	$\mathcal{P}_3$	$\cdots$	$\mathcal{P}_{305}$	$\mathcal{P}_{306}$	$\mathcal{P}_{307}$
$\mathfrak{L}_1$	1	1	0	$\cdots$	0	1	0
$\mathfrak{L}_2$	0	1	1	$\cdots$	0	0	1
$\mathfrak{L}_3$	1	0	1	$\cdots$	0	0	0
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
$\mathfrak{L}_{305}$	0	0	0	$\cdots$	1	1	0
$\mathfrak{L}_{306}$	0	0	0	$\cdots$	0	1	1
$\mathfrak{L}_{307}$	1	0	0	$\cdots$	1	0	1

Let

$\varpi = [0,0,0,\dots,0]$ $\phi = [6,6,6,\dots,6]$ $\gamma = [12,12,12,\dots,12]$

$\sigma = [1,1,1,\dots,1]$ $f = [7,7,7,\dots,7]$ $\delta = [13,13,13,\dots,13]$

$\mu = [2,2,2,\dots,2]$ $\mathfrak{P} = [8,8,8,\dots,8]$ $\varepsilon = [14,14,14,\dots,14]$

$g = [3,3,3,\dots,3]$ $\omega = [9,9,9,\dots,9]$ $\eta = [15,15,15,\dots,15]$

$\lambda = [4,4,4,\dots,4]$ $\alpha = [10,10,10,\dots 10]$ $\vartheta = [16,16,16,\dots,16]$

$\varphi = [5,5,5,\dots,5]$ $\beta = [11,11,11,\dots,11]$

$\mathfrak{i} = 1,2,3,4,5,6,\dots,307$

Table 4

$\boldsymbol{k_{\mathfrak{i}} = \varpi + \mathfrak{L}_{\mathfrak{i}}}$ **, That is ,**

$\boldsymbol{k_1 = 1 + 0 = 1 \quad or \quad k_1 = 1 + 1 = 2}$

k_1	2	2	1	…	1	2	1
k_2	1	2	2	…	1	1	2
k_3	2	1	2	…	1	1	1
$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$	$\vdots$
k_{305}	1	1	1	…	2	2	1
k_{306}	1	1	1	…	1	2	2
k_{307}	2	1	1	…	2	1	2

Table 5

$\boldsymbol{m_i = \sigma + \mathfrak{L}_i}$, **That is ,**
$\boldsymbol{m_1 = 2 + 0 = 2 \quad or \quad m_1 = 2 + 1 = 3}$

m_1	3	3	2	…	2	3	2
m_2	2	3	3	…	2	2	3
m_3	3	2	3	…	2	2	2
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
m_{305}	2	2	2	…	3	3	2
m_{306}	2	2	2	…	2	3	3
m_{307}	3	2	2	…	3	2	3

Table 6

$\boldsymbol{n_i = \mu + \mathfrak{L}_i}$, **That is ,**
$\boldsymbol{n_1 = 3 + 0 = 3 \quad or \quad n_1 = 3 + 1 = 4}$

n_1	4	4	3	…	3	4	3
n_2	3	4	4	…	3	3	4
n_3	4	3	4	…	3	3	3
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
n_{305}	3	3	3	…	4	4	3
n_{306}	3	3	3	…	3	4	4
n_{307}	4	3	3	…	4	3	4

Table 7

$\boldsymbol{r_i = g + \mathfrak{L}_i}$, **That is ,**
$\boldsymbol{r_1 = 4 + 0 = 4 \quad or \quad r_1 = 4 + 1 = 5}$

r_1	5	5	4	…	4	5	4
r_2	4	5	5	…	4	4	5
r_3	5	4	5	…	4	4	4
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
r_{305}	4	4	4	…	5	5	1
r_{306}	4	4	4	…	4	5	5
r_{307}	5	4	4	…	5	4	5

Table 8

$\boldsymbol{u_i = \lambda + \mathfrak{L}_i}$, **That is ,**

$\boldsymbol{u_1 = 5 + 0 = 5 \;\; or \;\; u_1 = 5 + 1 = 6}$

u_1	6	6	5	…	5	6	5
u_2	5	6	6	…	5	5	6
u_3	6	5	6	…	5	5	5
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
u_{305}	5	5	5	…	6	6	5
u_{306}	5	5	5	…	5	6	6
u_{307}	6	5	5	…	6	5	6

Table 9

$\boldsymbol{v_i = \phi + \mathfrak{L}_i}$, **That is ,**

$\boldsymbol{v_1 = 6 + 0 = 6 \;\; or \;\; v_1 = 6 + 1 = 7}$

v_1	7	7	6	…	6	7	6
v_2	6	7	7	…	6	6	7
v_3	7	6	7	…	6	6	6
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
v_{305}	6	6	6	…	7	7	6
v_{306}	6	6	6	…	6	7	7
v_{307}	7	6	6	…	7	6	7

Table 10

$\boldsymbol{w_i = \not{f} + \mathfrak{L}_i}$, **That is ,**

$\boldsymbol{w_1 = 7 + 0 = 7 \;\; or \;\; w_1 = 7 + 1 = 8}$

w_1	8	8	7	…	7	8	7
w_2	7	8	8	…	7	7	8
w_3	8	7	8	…	7	7	7
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
w_{305}	7	7	7	…	8	8	7
w_{306}	7	7	7	…	7	8	8
w_{307}	8	7	7	…	8	7	8

Table 11

$\mathcal{y}_i = \mathfrak{P} + \mathfrak{L}_i$, **That is ,**

$\mathcal{y}_1 = 8 + 0 = 8 \ \ or \ \ \mathcal{y}_1 = 8 + 1 = 9$

$\mathcal{y}_1$	9	9	8	…	8	1	8
$\mathcal{y}_2$	8	9	9	…	8	8	9
$\mathcal{y}_3$	9	8	9	…	8	8	8
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{y}_{305}$	8	8	8	…	9	9	8
$\mathcal{y}_{306}$	8	8	8	…	8	9	9
$\mathcal{y}_{307}$	9	8	8	…	9	8	9

Table 12

$x_i = \omega + \mathfrak{L}_i$, **That is ,**

$x_1 = 9 + 0 = 9 \ or \ \ x_1 = 9 + 1 = 10$

x_1	10	10	9	…	9	10	9
x_2	9	10	10	…	9	9	10
x_3	10	9	10	…	9	9	9
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
x_{305}	9	9	9	…	10	10	9
x_{306}	9	9	9	…	9	10	10
x_{307}	10	9	9	…	10	9	10

Table 13

$\mathcal{F}_i = \alpha + \mathfrak{L}_i$, **That is ,**

$\mathcal{F}_1 = 10 + 0 = 10 \ or \ \ \mathcal{F}_1 = 10 + 1 = 11$

$\mathcal{F}_1$	11	11	10	…	10	11	10
$\mathcal{F}_2$	10	11	11	…	10	10	11
$\mathcal{F}_3$	11	10	11	…	10	10	10
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{F}_{305}$	10	10	10	…	11	11	10
$\mathcal{F}_{306}$	10	10	10	…	10	11	11
$\mathcal{F}_{307}$	11	10	10	…	11	10	11

Table 14

$\boldsymbol{q_i = \beta + \mathfrak{L}_i}$ **, That is ,**

$\boldsymbol{q_1 = 11 + 0 = 11 \; or \;\; q_1 = 11 + 1 = 12}$

q_1	12	12	11	…	11	12	11
q_2	11	12	12	…	11	11	12
q_3	12	11	12	…	11	11	11
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
q_{305}	11	11	11	…	12	12	11
q_{306}	11	11	11	…	11	12	12
q_{307}	12	11	11	…	12	11	12

Table 15

$\boldsymbol{\mathcal{O}_i = \gamma + \mathfrak{L}_i}$ **, That is ,**

$\boldsymbol{\mathcal{O}_1 = 12 + 0 = 12 \; or \;\; \mathcal{O}_1 = 12 + 1 = 13}$

$\mathcal{O}_1$	13	13	12	…	12	13	12
$\mathcal{O}_2$	12	13	13	…	12	12	13
$\mathcal{O}_3$	13	12	13	…	12	12	12
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{O}_{305}$	12	12	12	…	13	13	12
$\mathcal{O}_{306}$	12	12	12	…	12	13	13
$\mathcal{O}_{307}$	13	12	12	…	13	12	13

Table 16

$\boldsymbol{\mathcal{C}_i = \delta + \mathfrak{L}_i}$ **, That is ,**

$\boldsymbol{\mathcal{C}_1 = 13 + 0 = 13 \; or \;\; \mathcal{C}_1 = 13 + 1 = 14}$

$\mathcal{C}_1$	14	14	13	…	13	14	13
$\mathcal{C}_2$	13	14	14	…	13	13	14
$\mathcal{C}_3$	14	13	14	…	13	13	13
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{C}_{305}$	13	13	13	…	14	14	13
$\mathcal{C}_{306}$	13	13	13	…	13	14	14
$\mathcal{C}_{307}$	14	13	13	…	14	13	14

Table 17

$\mathcal{A}_{\mathrm{i}} = \varepsilon + \mathfrak{L}_{\mathrm{i}}$, **That is ,**

$\mathcal{A}_1 = 14 + 0 = 14 \; or \;\; \mathcal{A}_1 = 14 + 1 = 15$

$\mathcal{A}_1$	15	15	14	…	14	15	14
$\mathcal{A}_2$	14	15	15	…	14	14	15
$\mathcal{A}_3$	15	14	15	…	14	14	14
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{A}_{305}$	14	14	14	…	15	15	14
$\mathcal{A}_{306}$	14	14	14	…	14	15	15
$\mathcal{A}_{307}$	15	14	14	…	15	14	15

Table 18

$\mathcal{D}_{\mathrm{i}} = \eta + \mathfrak{L}_{\mathrm{i}}$, **That is ,**

$\mathcal{D}_1 = 15 + 0 = 15 \; or \;\; \mathcal{D}_1 = 15 + 1 = 16$

$\mathcal{D}_1$	16	16	15	…	15	16	15
$\mathcal{D}_2$	15	16	16	…	15	15	16
o_3	16	15	16	…	15	15	15
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{D}_{305}$	15	15	15	…	16	16	15
$\mathcal{D}_{306}$	15	15	15	…	15	16	16
$\mathcal{D}_{307}$	16	15	15	…	16	15	16

Table 19

$\mathcal{T}_{\mathrm{i}} = \vartheta + \mathfrak{L}_{\mathrm{i}}$, **That is ,**

$\mathcal{T}_1 = 16 + 0 = 16 \; or \;\; \mathcal{T}_1 = 16 + 1 = 0$

$\mathcal{T}_1$	0	0	16	…	16	0	16
$\mathcal{T}_2$	16	0	0	…	16	16	0
$\mathcal{T}_3$	0	16	0	…	16	16	16
⋮	⋮	⋮	⋮	⋮	⋮	⋮	⋮
$\mathcal{T}_{305}$	16	16	16	...	0	0	16
$\mathcal{T}_{306}$	16	16	16	…	16	0	0
$\mathcal{T}_{307}$	0	16	16	…	0	16	0

The residual vectors of code C are created combination of $\varpi, \sigma, \mu, g, \lambda, \varphi, \phi, f, \mathfrak{P}, \omega, \alpha, \beta, \gamma, \delta, \varepsilon, \eta, \vartheta, \mathfrak{L}_i, k_i, m_i, n_i, r_i, u_i, v_i, w_i, y_i, x_i,$
$\mathcal{F}_i, q_i, \mathcal{O}_i, \mathcal{K}_i, \mathcal{A}_i, \mathcal{D}_i$ and $\mathcal{T}_i$
where $i = 1,2, \ldots, 307$, Note that $d(\mathfrak{L}_i, \mathfrak{L}_j)$ = number of points on precisely one of the $\mathfrak{L}_i$ or $\mathfrak{L}_j$.
And after that

Table 20
the minimum distance d of a non-trivial code C.

$d(\varpi, \mathfrak{L}_i) = 18$	$d(\mathfrak{L}_i, k_i) = 307$	$d(k_i, m_i) = 307$	$d(m_i, k_i) = 307$
$d(\sigma, \mathfrak{L}_i) = 289$	$d(\mathfrak{L}_i, m_i) = 307$	$d(k_i, n_i) = 307$	$d(m_i, n_i) = 307$
$d(\mu, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, n_i) = 307$	$d(k_i, r_i) = 307$	$d(m_i, r_i) = 307$
$d(g, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, r_i) = 307$	$d(k_i, u_i) = 307$	$d(m_i, u_i) = 307$
$d(\lambda, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, u_i) = 307$	$d(k_i, v_i) = 307$	$d(m_i, v_i) = 307$
$d(\varphi, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, v_i) = 307$	$d(k_i, w_i) = 307$	$d(m_i, w_i) = 307$
$d(\phi, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, w_i) = 307$	$d(k_i, y_i) = 307$	$d(m_i, y_i) = 307$
$d(f, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, y_i) = 307$	$d(k_i, x_i) = 307$	$d(m_i, x_i) = 307$
$d(\mathfrak{P}, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, x_i) = 307$	$d(k_i, \mathcal{F}_i) = 307$	$d(m_i, \mathcal{F}_i) = 307$
$d(\omega, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{F}_i) = 307$	$d(k_i, q_i) = 307$	$d(m_i, q_i) = 307$
$d(\alpha, \ell_i) = 307$	$d(\mathfrak{L}_i, q_i) = 307$	$d(k_i, \mathcal{O}_i) = 307$	$d(m_i, \mathcal{O}_i) = 307$
$d(\beta, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{O}_i) = 307$	$d(k_i, \mathcal{C}_i) = 307$	$d(m_i, \mathcal{C}_i) = 307$
$d(\gamma, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{C}_i) = 307$	$d(k_i, \mathcal{A}_i) = 307$	$d(m_i, \mathcal{A}_i) = 307$
$d(\delta, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{A}_i) = 307$	$d(k_i, \mathcal{D}_i) = 307$	$d(m_i, \mathcal{D}_i) = 307$
$d(\varepsilon, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{D}_i) = 307$	$d(k_i, \mathcal{T}_i) = 307$	$d(m_i, \mathcal{T}_i) = 307$
$d(\eta, \mathfrak{L}_i) = 307$	$d(\mathfrak{L}_i, \mathcal{T}_i) = 307$		
$d(\vartheta, \mathfrak{L}_i) = 307$			

$d(n_i, k_i) = 307$	$d(r_i, k_i) = 307$	$d(u_i, k_i) = 307$	$d(v_i, k_i) = 307$	$d(w_i, k_i) = 307$
$d(n_i, m_i) = 307$	$d(r_i, m_i) = 307$	$d(u_i, m_i) = 307$	$d(v_i, m_i) = 307$	$d(w_i, m_i) = 307$
$d(n_i, r_i) = 307$	$d(r_i, n_i) = 307$	$d(u_i, n_i) = 307$	$d(v_i, n_i) = 307$	$d(w_i, n_i) = 307$
$d(n_i, u_i) = 307$	$d(r_i, u_i) = 307$	$d(u_i, r_i) = 307$	$d(v_i, r_i) = 307$	$d(w_i, r_i) = 307$
$d(n_i, v_i) = 307$	$d(r_i, v_i) = 307$	$d(u_i, v_i) = 307$	$d(v_i, u_i) = 307$	$d(w_i, u_i) = 307$
$d(n_i, w_i) = 307$	$d(r_i, w_i) = 307$	$d(u_i, w_i) = 307$	$d(v_i, w_i) = 307$	$d(w_i, v_i) = 307$
$d(n_i, y_i) = 307$	$d(r_i, y_i) = 307$	$d(u_i, y_i) = 307$	$d(v_i, y_i) = 307$	$d(w_i, y_i) = 307$
$d(n_i, x_i) = 307$	$d(r_i, x_i) = 307$	$d(u_i, x_i) = 307$	$d(v_i, x_i) = 307$	$d(w_i, x_i) = 307$

$d(n_i,\mathcal{F}_i)=307$	$d(r_i,\mathcal{F}_i)=307$	$d(u_i,\mathcal{F}_i)=307$	$d(v_i,\mathcal{F}_i)=307$	$d(w_i,\mathcal{F}_i)=307$
$d(n_i,q_i)=307$	$d(r_i,q_i)=307$	$d(u_i,q_i)=307$	$d(v_i,q_i)=307$	$d(w_i,q_i)=307$
$d(n_i,\mathcal{O}_i)=307$	$d(r_i,\mathcal{O}_i)=307$	$d(u_i,\mathcal{O}_i)=307$	$d(v_i,\mathcal{O}_i)=307$	$d(w_i,\mathcal{O}_i)=307$
$d(n_i,\mathcal{C}_i)=307$	$d(r_i,\mathcal{C}_i)=307$	$d(u_i,\mathcal{C}_i)=307$	$d(v_i,\mathcal{C}_i)=307$	$d(w_i,\mathcal{C}_i)=307$
$d(n_i,\mathcal{A}_i)=307$	$d(r_i,\mathcal{A}_i)=307$	$d(u_i,\mathcal{A}_i)=307$	$d(v_i,\mathcal{A}_i)=307$	$d(w_i,\mathcal{A}_i)=307$
$d(n_i,\mathcal{D}_i)=307$	$d(r_i,\mathcal{D}_i)=307$	$d(u_i,\mathcal{D}_i)=307$	$d(v_i,\mathcal{D}_i)=307$	$d(w_i,\mathcal{D}_i)=307$
$d(n_i,\mathcal{T}_i)=307$	$d(r_i,\mathcal{T}_i)=307$	$d(u_i,\mathcal{T}_i)=307$	$d(v_i,\mathcal{T}_i)=307$	$d(w_i,\mathcal{T}_i)=307$

$d(y_i,k_i)=307$	$d(x_i,k_i)=307$	$d(\mathcal{F}_i,k_i)=307$	$d(\mathcal{D}_i,k_i)=307$	$d(q_i,k_i)=307$
$d(y_i,m_i)=307$	$d(x_i,m_i)=307$	$d(\mathcal{F}_i,m_i)=307$	$d(\mathcal{D}_i,m_i)=307$	$d(q_i,m_i)=307$
$d(y_i,n_i)=307$	$d(x_i,n_i)=307$	$d(\mathcal{F}_i,n_i)=307$	$d(\mathcal{D}_i,n_i)=307$	$d(q_i,n_i)= 307$
$d(y_i,r_i)=307$	$d(x_i,r_i)=307$	$d(\mathcal{F}_i,r_i)=307$	$d(\mathcal{D}_i,r_i)=307$	$d(q_i,r_i)= 307$
$d(y_i,u_i)=307$	$d(x_i,u_i)=307$	$d(\mathcal{F}_i,u_i)=307$	$d(\mathcal{D}_i,u_i)=307$	$d(q_i,u_i)= 307$
$d(y_i,v_i)=307$	$d(x_i,v_i)=307$	$d(\mathcal{F}_i,v_i)=307$	$d(\mathcal{D}_i,v_i)=307$	$d(q_i,v_i)= 307$
$d(y_i,w_i)=307$	$d(x_i,w_i)=307$	$d(\mathcal{F}_i,w_i)=307$	$d(\mathcal{D}_i,w_i)=307$	$d(q_i,w_i)= 307$
$d(y_i,x_i)=307$	$d(x_i,y_i)=307$	$d(\mathcal{F}_i,y_i)=307$	$d(\mathcal{D}_i,y_i)=307$	$d(q_i,y_i)= 307$
$d(y_i,\mathcal{F}_i)=307$	$d(x_i,\mathcal{F}_i)=307$	$d(\mathcal{F}_i,x_i)=307$	$d(\mathcal{D}_i,x_i)=307$	$d(q_i,x_i)= 307$
$d(y_i,q_i)=307$	$d(x_i,q_i)=307$	$d(\mathcal{F}_i,q_i)=307$	$d(\mathcal{D}_i,\mathcal{F}_i)=307$	$d(q_i,\mathcal{F}_i)=307$
$d(y_i,\mathcal{O}_i)=307$	$d(x_i,\mathcal{O}_i)=307$	$d(\mathcal{F}_i,\mathcal{O}_i)=307$	$d(\mathcal{D}_i,q_i)=307$	$d(q_i,\mathcal{O}_i)=307$
$d(y_i,\mathcal{C}_i)=307$	$d(x_i,\mathcal{C}_i)=307$	$d(\mathcal{F}_i,\mathcal{C}_i)=307$	$d(\mathcal{D}_i,\mathcal{O}_i)=307$	$d(q_i,\mathcal{C}_i)=307$
$d(y_i,\mathcal{A}_i)=307$	$d(x_i,\mathcal{A}_i)=307$	$d(\mathcal{F}_i,\mathcal{A}_i)=307$	$d(\mathcal{D}_i,\mathcal{C}_i)=307$	$d(q_i,\mathcal{A}_i)=307$
$d(y_i,\mathcal{D}_i)=307$	$d(x_i,\mathcal{D}_i)=307$	$d(\mathcal{F}_i,\mathcal{D}_i)=307$	$d(\mathcal{D}_i,\mathcal{A}_i)=307$	$d(q_i,\mathcal{D}_i)=307$
$d(y_i,\mathcal{T}_i)=307$	$d(x_i,\mathcal{T}_i)=307$	$d(\mathcal{F}_i,\mathcal{T}_i)=307$	$d(\mathcal{D}_i,\mathcal{T}_i)=307$	$d(q_i,\mathcal{T}_i)=307$

$d(\mathcal{O}_i,k_i)=307$	$d(\mathcal{C}_i,k_i)=307$	$d(\mathcal{A}_i,k_i)=307$	$d(\mathcal{T}_i,k_i)=307$
$d(\mathcal{O}_i,m_i)=307$	$d(\mathcal{C}_i,m_i)=307$	$d(\mathcal{A}_i,m_i)=307$	$d(\mathcal{T}_i,m_i)=307$
$d(\mathcal{O}_i,n_i)=307$	$d(\mathcal{C}_i,n_i)=307$	$d(\mathcal{A}_i,n_i)=307$	$d(\mathcal{T}_i,n_i)=307$
$d(\mathcal{O}_i,r_i)=307$	$d(\mathcal{C}_i,r_i)=307$	$d(\mathcal{A}_i,r_i)=307$	$d(\mathcal{T}_i,r_i)=307$
$d(\mathcal{O}_i,u_i)=307$	$d(\mathcal{C}_i,u_i)=307$	$d(\mathcal{A}_i,u_i)=307$	$d(\mathcal{T}_i,u_i)=307$
$d(\mathcal{O}_i,v_i)=307$	$d(\mathcal{C}_i,v_i)=307$	$d(\mathcal{A}_i,v_i)=307$	$d(\mathcal{T}_i,v_i)=307$
$d(\mathcal{O}_i,w_i)=307$	$d(\mathcal{C}_i,w_i)=307$	$d(\mathcal{A}_i,w_i)=307$	$d(\mathcal{T}_i,w_i)=307$
$d(\mathcal{O}_i,y_i)=307$	$d(\mathcal{C}_i,y_i)=307$	$d(\mathcal{A}_i,y_i)=307$	$d(\mathcal{T}_i,y_i)=307$
$d(\mathcal{O}_i,x_i)=307$	$d(\mathcal{C}_i,x_i)=307$	$d(\mathcal{A}_i,x_i)=307$	$d(\mathcal{T}_i,x_i)=307$

$d(\mathcal{O}_i,\mathcal{F}_i)=307$	$d(\mathcal{C}_i,\mathcal{F}_i)=307$	$d(\mathcal{A}_i,\mathcal{F}_i)=307$	$d(\mathcal{T}_i,\mathcal{F}_i)=307$
$d(\mathcal{O}_i,q_i)=307$	$d(\mathcal{C}_i,q_i)=307$	$d(\mathcal{A}_i,q_i)=307$	$d(\mathcal{T}_i,q_i)=307$
$d(\mathcal{O}_i,\mathcal{C}_i)=307$	$d(\mathcal{C}_i,\mathcal{O}_i)=307$	$d(\mathcal{A}_i,\mathcal{O}_i)=307$	$d(\mathcal{T}_i,\mathcal{O}_i)=307$
$d(\mathcal{O}_i,\mathcal{A}_i)=307$	$d(\mathcal{C}_i,\mathcal{A}_i)=307$	$d(\mathcal{A}_i,\mathcal{C}_i)=307$	$d(\mathcal{T}_i,\mathcal{C}_i)=307$
$d(\mathcal{O}_i,\mathcal{D}_i)=307$	$d(\mathcal{C}_i,\mathcal{D}_i)=307$	$d(\mathcal{A}_i,\mathcal{D}_i)=307$	$d(\mathcal{T}_i,\mathcal{A}_i)=307$
$d(\mathcal{O}_i,\mathcal{T}_i)=307$	$d(\mathcal{C}_i,\mathcal{T}_i)=307$	$d(\mathcal{A}_i,\mathcal{T}_i)=307$	$d(\mathcal{T}_i,\mathcal{D}_i)=307$

$d(\sigma,\varpi)=307$	$d(\mu,\varpi)=307$	$d(g,\varpi)=307$	$d(\lambda,\varpi)=307$	$d(\varphi,\varpi)=307$
$d(\sigma,\mu)=307$	$d(\mu,\sigma)=307$	$d(g,\sigma)=307$	$d(\lambda,\sigma)=307$	$d(\varphi,\sigma)=307$
$d(\sigma,g)=307$	$d(\mu,g)=307$	$d(g,\mu)=307$	$d(\lambda,\mu)=307$	$d(\varphi,\mu)=307$
$d(\sigma,\lambda)=307$	$d(\mu,\lambda)=307$	$d(g,\lambda)=307$	$d(\lambda,g)=307$	$d(\varphi,g)=307$
$d(\sigma,\varphi)=307$	$d(\mu,\varphi)=307$	$d(g,\varphi)=307$	$d(\lambda,\varphi)=307$	$d(\varphi,\lambda)=307$
$d(\sigma,\phi)=307$	$d(\mu,\phi)=307$	$d(g,\phi)=307$	$d(\lambda,\phi)=307$	$d(\varphi,\phi)=307$
$d(\sigma,\not{f})=307$	$d(\mu,\not{f})=307$	$d(g,\not{f})=307$	$d(\lambda,\not{f})=307$	$d(\varphi,\not{f})=307$
$d(\sigma,\mathfrak{P})=307$	$d(\mu,\mathfrak{P})=307$	$d(g,\mathfrak{P})=307$	$d(\lambda,\mathfrak{P})=307$	$d(\varphi,\mathfrak{P})=307$
$d(\sigma,\omega)=307$	$d(\mu,\omega)=307$	$d(g,\omega)=307$	$d(\lambda,\omega)=307$	$d(\varphi,\omega)=307$
$d(\sigma,\alpha)=307$	$d(\mu,\alpha)=307$	$d(g,\alpha)=307$	$d(\lambda,\alpha)=307$	$d(\varphi,\alpha)=307$
$d(\sigma,\beta)=307$	$d(\mu,\beta)=307$	$d(g,\beta)=307$	$d(\lambda,\beta)=307$	$d(\varphi,\beta)=307$
$d(\sigma,\gamma)=307$	$d(\mu,\gamma)=307$	$d(g,\gamma)=307$	$d(\lambda,\gamma)=307$	$d(\varphi,\gamma)=307$
$d(\sigma,\delta)=307$	$d(\mu,\delta)=307$	$d(g,\delta)=307$	$d(\lambda,\delta)=307$	$d(\varphi,\delta)=307$
$d(\sigma,\varepsilon)=307$	$d(\mu,\varepsilon)=307$	$d(g,\varepsilon)=307$	$d(\lambda,\varepsilon)=307$	$d(\varphi,\varepsilon)=307$
$d(\sigma,\eta)=307$	$d(\mu,\eta)=307$	$d(g,\eta)=307$	$d(\lambda,\eta)=307$	$d(\varphi,\eta)=307$
$d(\sigma,\vartheta)=307$	$d(\mu,\vartheta)=307$	$d(g,\vartheta)=307$	$d(\lambda,\vartheta)=307$	$d(\varphi,\vartheta)=307$

$d(\phi,\varpi)=307$	$d(\not{f},\varpi)=307$	$d(\mathfrak{P},\varpi)=307$	$d(\omega,\varpi)=307$	$d(\alpha,\varpi)=307$
$d(\phi,\sigma)=307$	$d(\not{f},\sigma)=307$	$d(\mathfrak{P},\sigma)=307$	$d(\omega,\sigma)=307$	$d(\alpha,\sigma)=307$
$d(\phi,\mu)=307$	$d(\not{f},\mu)=307$	$d(\mathfrak{P},\mu)=307$	$d(\omega,\mu)=307$	$d(\alpha,\mu)=307$
$d(\phi,g)=307$	$d(\not{f},g)=307$	$d(\mathfrak{P},g)=307$	$d(\omega,g)=307$	$d(\alpha,g)=307$
$d(\phi,\lambda)=307$	$d(\not{f},\lambda)=307$	$d(\mathfrak{P},\lambda)=307$	$d(\omega,\lambda)=307$	$d(\alpha,\lambda)=307$
$d(\phi,\varphi)=307$	$d(\not{f},\varphi)=307$	$d(\mathfrak{P},\varphi)=307$	$d(\omega,\varphi)=307$	$d(\alpha,\varphi)=307$
$d(\phi,S)=307$	$d(\not{f},\phi)=307$	$d(\mathfrak{P},\phi)=307$	$d(\omega,\phi)=307$	$d(\alpha,\phi)=307$
$d(\phi,\mathfrak{P})=307$	$d(\not{f},\mathfrak{P})=307$	$d(\mathfrak{P},\not{f})=307$	$d(\omega,\not{f})=307$	$d(\alpha,\not{f})=307$
$d(\phi,\omega)=307$	$d(\not{f},\omega)=307$	$d(\mathfrak{P},\omega)=307$	$d(\omega,\mathfrak{P})=307$	$d(\alpha,\mathfrak{P})=307$

$d(\phi,\alpha)=307$	$d(\not{f},\alpha)=307$	$d(\mathfrak{P},\alpha)=307$	$d(\omega,\alpha)=307$	$d(\alpha,\omega)=307$
$d(\phi,\beta)=307$	$d(\not{f},\beta)=307$	$d(\mathfrak{P},\beta)=307$	$d(\omega,\beta)=307$	$d(\alpha,\beta)=307$
$d(\phi,\gamma)=307$	$d(\not{f},\gamma)=307$	$d(\mathfrak{P},\gamma)=307$	$d(\omega,\gamma)=307$	$d(\alpha,\gamma)=307$
$d(\phi,\delta)=307$	$d(\not{f},\delta)=307$	$d(\mathfrak{P},\delta)=307$	$d(\omega,\delta)=307$	$d(\alpha,\delta)=307$
$d(\phi,\varepsilon)=307$	$d(\not{f},\varepsilon)=307$	$d(\mathfrak{P},\varepsilon)=307$	$d(\omega,\varepsilon)=307$	$d(\alpha,\varepsilon)=307$
$d(\phi,\eta)=307$	$d(\not{f},B)=307$	$d(\mathfrak{P},\eta)=307$	$d(\omega,\eta)=307$	$d(\alpha,\eta)=307$
$d(\phi,\vartheta)=307$	$d(\not{f},\vartheta)=307$	$d(\mathfrak{P},\vartheta)=307$	$d(\omega,\vartheta)=307$	$d(\alpha,\vartheta)=307$

$d(\varpi,k_i)=307$	$d(\sigma,k_i)=18$	$d(\mu,k_i)=289$	$d(g,k_i)=307$
$d(\varpi,m_i)=307$	$d(\sigma,m_i)=307$	$d(\mu,m_i)=18$	$d(g,m_i)=289$
$d(\varpi,n_i)=307$	$d(\sigma,n_i)=307$	$d(\mu,n_i)=307$	$d(g,n_i)=18$
$d(\varpi,r_i)=307$	$d(\sigma,r_i)=307$	$d(\mu,r_i)=307$	$d(g,r_i)=307$
$d(\varpi,u_i)=307$	$d(\sigma,u_i)=307$	$d(\mu,u_i)=307$	$d(g,u_i)=307$
$d(\varpi,v_i)=307$	$d(\sigma,v_i)=307$	$d(\mu,v_i)=307$	$d(g,v_i)=307$
$d(\varpi,w_i)=307$	$d(\sigma,w_i)=307$	$d(\mu,w_i)=307$	$d(g,w_i)=307$
$d(\varpi,y_i)=307$	$d(\sigma,y_i)=307$	$d(\mu,y_i)=307$	$d(g,y_i)=307$
$d(\varpi,x_i)=307$	$d(\sigma,x_i)=307$	$d(\mu,x_i)=307$	$d(g,x_i)=307$
$d(\varpi,\mathcal{F}_i)=307$	$d(\sigma,\mathcal{F}_i)=307$	$d(\mu,\mathcal{F}_i)=307$	$d(g,\mathcal{F}_i)=307$
$d(\varpi,q_i)=307$	$d(\sigma,q_i)=307$	$d(\mu,q_i)=307$	$d(g,q_i)=307$
$d(\varpi,\mathcal{O}_i)=307$	$d(\sigma,\mathcal{O}_i)=307$	$d(\mu,\mathcal{O}_i)=307$	$d(g,\mathcal{O}_i)=307$
$d(\varpi,\mathcal{C}_i)=307$	$d(\sigma,\mathcal{C}_i)=307$	$d(\mu,\mathcal{C}_i)=307$	$d(g,\mathcal{C}_i)=307$
$d(\varpi,\mathcal{A}_i)=307$	$d(\sigma,\mathcal{A}_i)=307$	$d(\mu,\mathcal{A}_i)=307$	$d(g,\mathcal{A}_i)=307$
$d(\varpi,\mathcal{D}_i)=307$	$d(\sigma,\mathcal{D}_i)=307$	$d(\mu,\mathcal{D}_i)=307$	$d(g,\mathcal{D}_i)=307$
$d(\varpi,\mathcal{T}_i)=289$	$d(\sigma,\mathcal{T}_i)=307$	$d(\mu,\mathcal{T}_i)=307$	$d(g,\mathcal{T}_i)=307$

$d(\lambda,k_i)=307$	$d(\varphi,k_i)=307$	$d(\phi,k_i)=307$	$d(\not{f},k_i)=307$
$d(\lambda,m_i)=307$	$d(\varphi,m_i)=307$	$d(\phi,m_i)=307$	$d(\not{f}\ m_i)=307$
$d(\lambda,n_i)=289$	$d(\varphi,n_i)=307$	$d(\phi,n_i)=307$	$d(\not{f},n_i)=307$
$d(\lambda,r_i)=18$	$d(\varphi,r_i)=289$	$d(\phi,r_i)=307$	$d(\not{f},r_i)=307$
$d(\lambda,u_i)=307$	$d(\varphi,u_i)=18$	$d(\phi,u_i)=289$	$d(\not{f},u_i)=307$
$d(\lambda,v_i)=307$	$d(\varphi,v_i)=307$	$d(\phi,v_i)=18$	$d(\not{f},v_i)=289$
$d(\lambda,w_i)=307$	$d(\varphi,w_i)=307$	$d(\phi,w_i)=307$	$d(\not{f},w_i)=18$

$d(\lambda,\mathscr{y}_i)$ =307	$d(\varphi,\mathscr{y}_i)$ =307	$d(\phi,\mathscr{y}_i) = 307$	$d(\not{f},\mathscr{y}_i) = 307$
$d(\lambda,\mathscr{x}_i)$ =307	$d(\varphi,\mathscr{x}_i)$ =307	$d(\phi,\mathscr{x}_i) = 307$	$d(\not{f},\mathscr{x}_i) = 307$
$d(\lambda,\mathscr{F}_i)$ =307	$d(\varphi,\mathscr{F}_i)$ =307	$d(\phi,\mathscr{F}_i) = 307$	$d(\not{f},\mathscr{F}_i) = 307$
$d(\lambda,\mathscr{q}_i)$ =307	$d(\varphi,\mathscr{q}_i)$ =307	$d(\phi,\mathscr{q}_i) = 307$	$d(\not{f},\mathscr{q}_i) = 307$
$d(\lambda,\mathscr{O}_i)$ =307	$d(\varphi,\mathscr{O}_i)$ =307	$d(\phi,\mathscr{O}_i) = 307$	$d(\not{f},\mathscr{O}_i) = 307$
$d(\lambda,\mathscr{C}_i)$ =307	$d(\varphi,\mathscr{C}_i)$ =307	$d(\phi,\mathscr{C}_i) = 307$	$d(\not{f},\mathscr{C}_i) = 307$
$d(\lambda,\mathscr{A}_i)$ =307	$d(\varphi,\mathscr{A}_i)$ =307	$d(\phi,\mathscr{A}_i) = 307$	$d(\not{f},\mathscr{A}_i) = 307$
$d(\lambda,\mathscr{D}_i)$ =307	$d(\varphi,\mathscr{D}_i)$ =307	$d(\phi,\mathscr{D}_i) = 307$	$d(\not{f},\mathscr{D}_i) = 307$
$d(\lambda,\mathscr{T}_i)$ =307	$d(\varphi,\mathscr{T}_i)$ =307	$d(\phi,\mathscr{T}_i) = 307$	$d(\not{f},\mathscr{T}_i) = 307$

$d(\mathfrak{P},\mathscr{k}_i)$ =307	$d(\omega,\mathscr{k}_i)$ =307	$d(\alpha,\mathscr{k}_i)$ =307	$d(\beta,\mathscr{k}_i)$ =307
$d(\mathfrak{P},\mathscr{m}_i)$ =307	$d(\omega,\mathscr{m}_i)$ =307	$d(\alpha,\mathscr{m}_i)$ =307	$d(\beta,\mathscr{m}_i)$ =307
$d(\mathfrak{P},\mathscr{n}_i)$ =307	$d(\omega,\mathscr{n}_i)$ =307	$d(\alpha,\mathscr{n}_i)$ =307	$d(\beta,\mathscr{n}_i)$ =307
$d(\mathfrak{P},\mathscr{r}_i)$ =307	$d(\omega,\mathscr{r}_i)$ =307	$d(\alpha,\mathscr{r}_i)$ =307	$d(\beta,\mathscr{r}_i)$ =307
$d(\mathfrak{P},\mathscr{u}_i)$ =307	$d(\omega,\mathscr{u}_i)$ =307	$d(\alpha,\mathscr{u}_i)$ =307	$d(\beta,\mathscr{u}_i)$ =307
$d(\mathfrak{P},\mathscr{v}_i)$ =307	$d(\omega,\mathscr{v}_i)$ =307	$d(\alpha,\mathscr{v}_i)$ =307	$d(\beta,\mathscr{v}_i)$ =307
$d(\mathfrak{P},\mathscr{w}_i)$ =289	$d(\omega,\mathscr{w}_i)$ =307	$d(\alpha,\mathscr{w}_i)$ =307	$d(\beta,\mathscr{w}_i)$ =307
$d(\mathfrak{P},\mathscr{y}_i)$ =18	$d(\omega,\mathscr{y}_i)$ =289	$d(\alpha,\mathscr{y}_i)$ =307	$d(\beta,\mathscr{y}_i)$ =307
$d(\mathfrak{P},\mathscr{x}_i)$ =307	$d(\omega,\mathscr{x}_i)$ =18	$d(\alpha,\mathscr{x}_i)$ =289	$d(\beta,\mathscr{x}_i)$ =307
$d(\mathfrak{P},\mathscr{F}_i)$ =307	$d(\omega,\mathscr{F}_i)$ =307	$d(\alpha,\mathscr{F}_i)$ =18	$d(\beta,\mathscr{F}_i)$ =289
$d(\mathfrak{P},\mathscr{q}_i)$ =307	$d(\omega,\mathscr{q}_i)$ =307	$d(\alpha,\mathscr{q}_i)$ =307	$d(\beta,\mathscr{q}_i)$ =18
$d(\mathfrak{P},\mathscr{O}_i)$ =307	$d(\omega,\mathscr{O}_i)$ =307	$d(\alpha,\mathscr{O}_i)$ =307	$d(\beta,\mathscr{O}_i)$ =307
$d(\mathfrak{P},\mathscr{C}_i)$ =307	$d(\omega,\mathscr{C}_i)$ =307	$d(\alpha,\mathscr{C}_i)$ =307	$d(\beta,\mathscr{C}_i)$ =307
$d(\mathfrak{P},\mathscr{A}_i)$ =307	$d(\omega,\mathscr{A}_i)$ =307	$d(\alpha,\mathscr{A}_i)$ =307	$d(\beta,\mathscr{A}_i)$ =307
$d(\mathfrak{P},\mathscr{D}_i)$ =307	$d(\omega,\mathscr{D}_i)$ =307	$d(\alpha,\mathscr{D}_i)$ =307	$d(\beta,\mathscr{D}_i)$ =307
$d(\mathfrak{P},\mathscr{T}_i)$ =307	$d(\omega,\mathscr{T}_i)$ =307	$d(\alpha,\mathscr{T}_i)$ =307	$d(\beta,\mathscr{T}_i)$ =307

$d(\gamma,\mathscr{k}_i)$ =307	$d(\delta,\mathscr{k}_i)$ =307	$d(\varepsilon,\mathscr{k}_i)$ =307	$d(\eta,\mathscr{k}_i)$ =307
$d(\gamma,\mathscr{m}_i)$ =307	$d(\delta,\mathscr{m}_i)$ =307	$d(\varepsilon,\mathscr{m}_i)$ =307	$d(\eta,\mathscr{m}_i)$ =307
$d(\gamma,\mathscr{n}_i)$ =307	$d(\delta,\mathscr{n}_i)$ =307	$d(\varepsilon,\mathscr{n}_i)$ =307	$d(\eta,\mathscr{n}_i)$ =307
$d(\gamma,\mathscr{r}_i)$ =307	$d(\delta,\mathscr{r}_i)$ =307	$d(\varepsilon,\mathscr{r}_i)$ =307	$d(\eta,\mathscr{r}_i)$ =307
$d(\gamma,\mathscr{u}_i)$ =307	$d(\delta,\mathscr{u}_i)$ =307	$d(\varepsilon,\mathscr{u}_i)$ =307	$d(\eta,\mathscr{u}_i)$ =307
$d(\gamma,\mathscr{v}_i)$ =307	$d(\delta,\mathscr{v}_i)$ =307	$d(\varepsilon,\mathscr{v}_i)$ =307	$d(\eta,\mathscr{v}_i)$ =307

$d(\gamma, \mathcal{w}_i)$ =307	$d(\delta, \mathcal{w}_i)$ =307	$d(\varepsilon, \mathcal{w}_i)$ =307	$d(\eta, \mathcal{w}_i)$ =307
$d(\gamma, \mathcal{y}_i)$ =307	$d(\delta, \mathcal{y}_i)$ =307	$d(\varepsilon, \mathcal{y}_i)$ =307	$d(\eta, \mathcal{y}_i)$ =307
$d(\gamma, \mathcal{x}_i)$ =307	$d(\delta, \mathcal{x}_i)$ =307	$d(\varepsilon, \mathcal{x}_i)$ =307	$d(\eta, \mathcal{x}_i)$ =307
$d(\gamma, \mathcal{F}_i)$ =307	$d(\delta, \mathcal{F}_i)$ =307	$d(\varepsilon, \mathcal{F}_i)$ =307	$d(\eta, \mathcal{F}_i)$ =307
$d(\gamma, \mathcal{q}_i)$ =289	$d(\delta, \mathcal{q}_i)$ =307	$d(\varepsilon, \mathcal{q}_i)$ =307	$d(\eta, \mathcal{q}_i)$ =307
$d(\gamma, \mathcal{O}_i)$ =18	$d(\delta, \mathcal{O}_i)$ =289	$d(\varepsilon, \mathcal{O}_i)$ =307	$d(\eta, \mathcal{O}_i)$ =307
$d(\gamma, \mathcal{C}_i)$ =307	$d(\delta, \mathcal{C}_i)$ =18	$d(\varepsilon, \mathcal{C}_i)$ =289	$d(\eta, \mathcal{C}_i)$ =307
$d(\gamma, \mathcal{A}_i)$ =307	$d(\delta, \mathcal{A}_i)$ =307	$d(\varepsilon, \mathcal{A}_i)$ =18	$d(\eta, \mathcal{A}_i)$ =289
$d(\gamma, \mathcal{D}_i)$ =307	$d(\delta, \mathcal{D}_i)$ =307	$d(\varepsilon, \mathcal{D}_i)$ =307	$d(\eta, \mathcal{D}_i)$ =18
$d(\gamma, \mathcal{T}_i)$ =307	$d(\delta, \mathcal{T}_i)$ =307	$d(\varepsilon, \mathcal{T}_i)$ =307	$d(\eta, \mathcal{T}_i)$ =307

In the inequality of theorem 1.2, we obtain $M = q^{k+1} = 17^4$ if we replace the values of $n = 307$, $d = 18$, $e = 8$
resulting in C is a $(307, 17^4, 18) - code$.

$$17^4 \left\{ \begin{matrix} \binom{307}{0} + \binom{307}{1}(17-1) + \binom{307}{2}(17-1)^2 + \binom{307}{3}(17-1)^3 + \binom{307}{4}(17-1)^4 \\ +\binom{307}{5}(17-1)^5 + \binom{307}{6}(17-1)^6 + \binom{307}{7}(17-1)^7 + \binom{307}{8}(17-1)^8 \end{matrix} \right\} \leq 17^{307}$$

Via Corollary1.3, therefore C is perfect

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