



## $i$ –Semi-Inter-Open Sets in Topological Spaces

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### Abstract

This work introduces a novel class of *semi-open* sets are called  $i$ -semi-inter open sets and semi-inter open sets in topological spaces.

A number of mathematical arguments have been used to thoroughly examine some of these sets' characteristics and their connections to other kinds of sets, including, for example we proved that in any topological space  $(X, \tau)$ : Each  $\alpha$  – open set is  $i$  – semi inter open set and semi-inter open set, each *semi-open* set is semi-inter open set and  $i$  – semi inter open set, each  $i$  – semi inter open set is  $i$ -open set and semi inter open set, each open set is semi inter open set and  $i$  – semi inter open set, however, generally speaking the converses are not necessary to be valid. Also, we defined the topologically extended and non-topologically extended property for  $i$ -semi-inter open sets, and we show that the family of  $i$ -semi inter open sets is not necessarily a topological space, with some characterizations as  $i$ -semi-inter limit points,  $i$ -semi-inter boundary of a set.

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### 1. Introduction

Levine, N. introduced the concept of *semi-open* sets in 1963 [7], and it enhanced numerous significant foundational notions of general topology. In 1965, Njastad, O. presented the idea of  $\alpha$ -open sets, a subclass of *semi-open* sets. Moreover, for every  $\tau \subset \tau^\alpha$  and  $\tau^\alpha$  is always a topology on  $X$ , where  $\tau^\alpha$  denotes the family of all  $\alpha$ -open sets of  $(X, \tau)$  (cf. [10], [11]) Askandar, S.W., [3] in 2012, offered the notion of  $i$ -open sets, which is a subclass of *semi-open* sets. Mohammed, A.A. and Abdullah, B.S., [9] in 2019, introduced the concept of *ii-open* sets and inter open sets. Ashaea, G. S. and Yousif, Y.Y., in [4], in 2021, introduced some types of mappings in bi-topological spaces. Yousif, Y. Y. and Hussein, N. A., in [17], in 2024, studied the  $\alpha$ - $\alpha$ -connected fibrewise topological spaces. Saleh, E.A. and Sulaiman, B.M., in [14], in 2023, introduced the concept of micro-grill-generalized open set. In our study, we use these classes mentioned above to compare and to find similar properties and characterization of  $i$ -semi-inter open sets. The class of  $i$ -semi-inter open sets perhaps to enter together with classes such as, pre-open sets [8], *semi-open* sets [7],  $\alpha$ -open sets [10],  $\beta$ -open sets [1], regular open sets [15], regular *semi-open* sets [5],  $b$ -open sets [6], pre semi open sets [8],  $\delta$ -open sets [16],  $\theta$ -open sets [16],  $i$ -open sets [3]. There are four parts in this work. In the first part, we recall that *semi-open* sets,  $\alpha$ -open sets,  $i$ -open sets and *ii-open* sets. In the second part, we define  $i$ -semi-inter open sets, and we provide numerous examples. Also, we study the relation and compare  $i$ -semi-inter open sets with *semi-open* sets,  $\alpha$ -open sets and  $i$ -open sets respectively. In the third part, we define the Topologically Extended for  $i$ -semi-inter open sets, Non- Topologically Extended for  $i$ -semi-inter open sets. Finally, in the fourth part, we study some properties of  $i$ -semi-inter open sets. Throughout this paper, we denote by  $TS$  a topological space. We denote the closure (respectively, the interior of a set  $K$ ) as  $CL(K)$ , (respectively,  $Int(K)$ ), where  $CL(K)$  is the intersection of all closed sets containing  $K$ ,  $Int(K)$  is the union of all open sets contained in  $K$  [12,13].

## 2. Preliminaries

This portion will provide numerous fundamental definitions for a variety of open (closed)sets in  $TS$ , also we review the fundamental theories of sets in  $TS$ .

**Definition 2.1.** A subset  $K$  of a  $TS (X, \tau)$  is said to be

- a- [7]"Semi- Open Set", for short ( $SOs$ ), if  $K \subseteq CL(Int(K))$ . The family of all semi-open sets is denoted by  $\tau^S$ .
- b- [10]" $\alpha$  –Open Set", for short ( $\alpha Os$ ), if satisfies the condition:  $K \subseteq Int (CL(Int(K)))$ . The family of all  $\alpha$  -open sets is denoted by  $\tau^\alpha$ .
- c- [3]" $i$  – Open Set", for short ( $IOs$ ), If an *open* set  $G \neq \emptyset, X$  exists, then  $K \subseteq CL(K \cap G)$ . The family of all  $i$ -open sets is denoted by  $\tau^I$
- d- [9]"Inter- Open Set", for short ( $INTOs$ ), If an *open* set  $G \neq \emptyset, X$  exists, then  $Int(K) = G$ .
- e- [9]"ii- open set", ( $IIOs$ ), if there exists an *open* set  $G \in O(X)$ , in which  $G \neq \emptyset, X$ ,  
 $K \subseteq CL(K \cap G), Int(K) = G$ . The family of all ii-open sets is denoted by  $\tau^{II}$  and it is always a topological space.

**Definition2.2.** Let  $K$  be a subset in a  $TS(X, \tau)$ , then:

An "Semi- interior"[7] [ respectively, " $\alpha$ - interior"[10] and " $i$ - interior"[2,3]) of a set  $K$  is the union of all  $SOs$  (respectively, ( $\alpha Os$ ) & ( $IOs$ ),) over  $X$  included in  $K$ , denoted by  $sInt (K)$  [ respectively,  $\alpha Int (K)$  &  $IInt (K)$  ).

"Semi-closed" for short ( $SCs$ ) [7] (respectively, " $\alpha$ -closed" for short ( $\alpha Cs$ ) [10] and " $i$  –closed set" for short ( $ICs$ ) [2,3]) is the whole complement of  $SOs$  (respectively,  $\alpha Os$  &  $IOs$  ).

The "Semi-closure"[7] (respectively " $\alpha$ - closure"[10] and " $i$ - closure"[2,3] of  $K$  is indicated by  $SCL (K)$  (respectively,  $\alpha Cl (K)$  &  $ICl (K)$ ) is the intersection of all  $SCs$  (respectively,  $\alpha Cs$  &  $ICs$ ) over  $X$  comprising  $K$ .

**Proposition2.3.** [3] Each  $Os$  is a  $IOs$ .

**Proposition2.4.** [3] Each  $Cs$  is a  $ICs$ .

**Proposition2.5.** [3] Each  $SOs$  is a  $IOs$ .

**Propositin2.6.** [10] Each  $\alpha Os$  is a  $SOs$ .

**Proposition2.7.** [3] Each  $\alpha Os$  is a  $IOs$ .

**Proposition2.8.** [7] Each  $Os$  is a  $SOs$ .

**Definition 2.9.** [12,13] Let  $K$  be a subset of  $X$ , and let  $(X, \tau)$  be a  $TS$ . The term "limit point of  $K$ " refers to a point  $x \in X$  if, for every *open* set  $G$  that contains  $x$ , then  $(K \cap G) \setminus \{x\} \neq \emptyset$ , "The derived set of  $K$ " is the set of all limit points of  $K$ , and it is represented by  $D(K)$ .  $K$  is closed if and only if  $D(K) \subseteq K$ .

**Definition.2.10.** [12,13] Let  $(X, \tau)$  be a  $TS$  and let  $K$  be a subset of  $X$ . The Interior of the complement of  $K$  is called the "Exterior of a set  $K$ ", and denoted by  $Ext(K)$ , and defined as:

$$Ext(K) = Int(K^C).$$

**Definition2.11.** [12,13] Let  $K$  be a subset of  $X$ , and let  $(X, \tau)$  be a  $TS$ . "The boundary of a set  $K$ " is indicated by  $b(K)$ , and defined as:  $b(K) = (Int(K) \cup Ext(K))^C$ .

## 3. $i$ -Semi-Inter- Open Sets

In this section, we define the concept of  $i$ -semi-inter *open* sets, also we study the relationship and compare  $i$ -semi-inter *open* sets with  $Os$ ,(resp.  $SOs, \alpha Os, IOs$ ).

**Definition.3.1.** A subset  $K$  of  $TS (X, \tau)$  is referred to as "Semi-Inter *Open* Set" (in short  $S. INTOs$ ) if there exists a *semi –open* set  $S \neq \emptyset, X$  in  $(X, \tau)$ , i.e.  $sInt(K) = S$ , where  $sInt(K)$  is the union of all *semi –open* sets which are included in  $K$ .( i.e.  $K$  is said to be semi-inter open set if  $sInt(K)$  is not a trivial semi-open set). We indicated by  $\tau^{SINT}$  to the entire family of semi-inter *open* sets in  $X$  with  $\emptyset$  and  $X$ .

The complement of semi –inter *open* set is known as "semi-inter closed set" (in short  $S. INTCs$ ). We indicated by  $\tau^{CSINT}$  to the family of all semi-inter closed sets in  $X$  with  $\emptyset$  and  $X$ .

**Definition.3.2.** If a subset  $K$  of a  $TS(X, \tau)$  meets the following conditions, it is referred to as a " $i$  -Semi-Inter *Open* Set" (abbreviated  $I.S. INTOs$ ):

- 1-  $K$  is  $i$ - *open* set.

2-  $K$  is semi-inter open set, i.e. there exists a semi-open set  $S \neq \emptyset, X$  in  $(X, \tau)$  with  $sInt(K) = S$ .

We denoted by  $\tau^{ISINT}$  to the family of all  $i$ -semi-inter open set with  $\emptyset$  and  $X$ .

$$\tau^{ISINT} = (\tau^I \cap \tau^{SINT}) \cup \{\emptyset, X\}.$$

The " $i$ -semi-inter closed set" (abbreviated  $I.S. INTCS$ ) is the complement of  $i$ -semi-inter open. The family of every  $i$ -semi-inter closed sets in  $X$  with  $\emptyset$  and  $X$  is represented by  $\tau^{CISINT}$ .

**Definition.3.3.** A subset  $K$  of a  $TS(X, \tau)$  is said to be " $i$ -Semi Inter Closed Set" (in short  $I.S. INTCS$ ) if it meets the requirements listed below:

1-  $K$  is  $i$ -closed set.

2-  $K$  is a semi-inter closed set, i.e. there exists a semi-closed set  $C \neq \emptyset, X$  in  $(X, \tau)$  with  $sCL(K) = C$ .

**Example.3.4.** Let  $X = \{p, q, u, v\}, \tau = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, X\}$

$$\tau^C = \{X, \{q, u, v\}, \{p, q\}, \{q\}, \phi\}$$

$$\tau^I = \{\phi, \{p\}, \{u\}, \{v\}, \{u, v\}, \{p, u, v\}, \{p, q\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{q, u\}, \{q, v\}, \{q, u, v\}, X\}$$

$$\tau^S = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, \{p, q\}, \{q, u, v\}, X\}$$

$$\tau^{SINT} = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, \{p, q\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, X\}$$

$$\tau^{ISINT} = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, \{p, q\}, \{p, u\}, \{p, q, u\}, \{p, v\}, \{p, q, v\}, \{q, u, v\}, X\}$$

$$\tau^{CISINT} = \{X, \{q, u, v\}, \{p, q\}, \{q\}, \{u, v\}, \{q, v\}, \{v\}, \{q, u\}, \{u\}, \{p\}, \emptyset\}$$

**Example.3.5.** Let  $X = \{p, q, u\}, \tau = \{\phi, \{p\}, \{q\}, \{p, q\}, X\}$

$$\tau^C = \{\phi, \{q, u\}, \{p, u\}, \{u\}, X\}$$

$$\tau^I = \{\phi, \{p\}, \{q\}, \{p, q\}, \{p, u\}, \{q, u\}, X\}$$

$$\tau^S = \{\phi, \{p\}, \{q\}, \{p, q\}, \{p, u\}, \{q, u\}, X\}$$

$$\tau^{SINT} = \{\phi, \{p\}, \{q\}, \{p, q\}, \{p, u\}, \{q, u\}, X\}$$

$$\tau^{ISINT} = \{\phi, \{p\}, \{q\}, \{p, q\}, \{p, u\}, \{q, u\}, X\}$$

**Remark.3.6.** The intersection of two  $i$ -semi-inter-open sets is not necessarily to be  $i$ -semi-inter-open set.

**In Example.3.5.** we see that  $\{p, u\}$  and  $\{q, u\}$  are  $I.S. INTOs$  but  $\{p, u\} \cap \{q, u\} = \{u\}$  is not  $I.S. INTOs$ .

**Theorem.3.7.** Each  $\alpha Os$  in any  $TS$  is  $I.S. INTOs$ .

Proof: Let  $K$  be  $\alpha Os$  in  $(X, \tau)$ , since each  $\alpha Os$  is a  $IOs$  (By the Proposion.2.7), we get  $K$  is  $IOs$

Also, since each  $\alpha Os$  is a  $SOs$  (By the Proposion.2.6)), we get  $K$  is  $SOs$ , Therefore,  $sInt(K) = K$ , Hence,  $K$  is  $S. INTOs$ ,

Then  $K$  is  $I.S. INTOs$ .

**It is not required for the opposite of "Theorem 3.7" to be true:**

**In Example.3.4.**  $\tau^\alpha = \{\emptyset, \{p\}, \{u, v\}, \{p, u, v\}, X\}, K = \{p, v\}$  is  $I.S. INTOs$  but it is not  $\alpha Os$ .

**Theorem.3.8.** Each  $SOs$  in any  $TS$  is  $S. INTOs$ .

Proof: Let  $K$  be  $SOs$  in  $(X, \tau)$ , we get  $sInt(K) = K$ , which is  $SOs$  (by suppose),

hence  $K$  is  $S.INTOs$ .

**It is not required for the opposite of “Theorem 3.8” to be true:**

**In Example.3.4.**  $K = \{p, q, v\}$  is  $S.INTOs$ , but it is not  $SOs$ .

**Theorem.3.9.** Each  $SOs$  in any  $TS$  is  $I.S.INTOs$ .

Proof: Let  $K$  be  $SOs$  in  $(X, \tau)$ , since each  $SOs$  is an  $IOs$  (By the Proposion.2.5.), we get  $K$  is  $IOs$ ,

Also,  $sInt(K) = K$ , (by suppose), we get  $K$  is  $S.INTOs$ , and hence,  $K$  is  $I.S.INTOs$ .

**It is not required for the opposite of “Theorem 3.9” to be true:**

**In Example.3.4.,**  $K = \{p, q, v\}$  is  $I.S.INTOs$ , but it is not  $SOs$ .

**Corollary.3.10.** Each  $\alpha Os$  set in any  $TS$  is  $S.INTOs$ .

**It is not required for the opposite of “Corollary.3.10” to be true:**

**In Example.3.4.**  $\tau^\alpha = \{\emptyset, \{p\}, \{u, v\}, \{p, u, v\}, X\}$ ,  $K = \{p, q\}$  is  $S.INTOs$ , but it is not  $\alpha Os$ .

**Remark.3.11.** Each  $I.S.INTOs$  in any  $TS$  is  $IOs$ .

**It is not required for the opposite of “Remark.3.11” to be true:**

**In Example.3.4.**  $K = \{u\}$  is  $IOs$ , but it is not  $I.S.INTOs$ .

**Remark.3.12.** Each  $I.S.INTOs$  in any  $TS$  is  $S.INTOs$ .

**It is not required for the opposite of “Remark.3.12” to be true:**

**Example.3.13.** Let  $X = \{p, q, u, v\}$ ,  $\tau = \{\phi, \{p\}, \{q, u, v\}, X\}$

$$\tau^C = \{X, \{q, u, v\}, \{p\}, \phi\}$$

$$\tau^I = \{\phi, \{p\}, \{q\}, \{u\}, \{v\}, \{q, u\}, \{q, v\}, \{q, u, v\}, \{u, v\}, X\}$$

$$\tau^S = \{\phi, \{p\}, \{q, u, v\}, X\}$$

$$\tau^{SINT} = \{\phi, \{p\}, \{p, q\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, \{q, u, v\}, X\}$$

$$\tau^{ISINT} = \{\phi, \{p\}, \{q, u, v\}, X\}$$

We can see that  $K = \{p, u, v\}$  is  $S.INTOs$ , but it is not  $I.S.INTOs$  because it is not  $IOs$ .

**Theorem.3.14.** Each  $Os$  in any  $TS$  is  $S.INTOs$ .

Proof: Let  $G$  be  $Os$  in  $(X, \tau)$ , since each  $Os$  is a  $SOs$  (by Proposion.2.8.). We get  $G$  is  $SOs$  and

$sInt(G) = G$ , hance  $G$  is  $S.INTOs$ .

**It is not required for the opposite of “Theorem.3.14” to be true:**

**In Example.3.13.**  $K = \{p, u\}$  is  $S.INTOs$ , but it is not  $Os$ .

**Theorem.3.15.** Each  $Os$  in any  $TS$  is  $I.S.INTOs$ .

Proof: Let  $G$  be  $O_s$  in a topological space  $(X, \tau)$ , (By Theorem.3.14.),

We get  $G$  is  $S.INTOs$ , since each  $O_s$  is an  $IOs$  (by Proposition.2.3.), We get  $G$  is  $IOs$ , hence,  $G$  is  $I.S.INTOs$ .

**It is not required for the opposite of “Theorem.3.15” to be true:**

**In Example.3.4.**  $K = \{p, q, u\}$  is  $I.S.INTOs$  but it is not  $O_s$ .

**Remark.3.16.** In general,  $IOs$  is not  $S.INTOs$ , also  $S.INTOs$  is not  $IOs$ .

**In Example.3.13.**  $K = \{p, q, v\}$  is  $S.INTOs$ , but it is not  $IOs$ , and  $M = \{u\}$  is  $IOs$  but is not  $S.INTOs$ .

#### 4. The Topologically Extended and Non-Topologically Extended Property for $i$ –Semi-Inter- open Sets.

**Definition.4.1.** It is said that a  $TS(X, \tau)$  is "Topologically Extended for  $i$ -semi-inter open sets" (in short  $T.E.I.S.INTOs$ ) if and only if  $(X, \tau^{ISINT})$  is a  $TS$ . Otherwise, it's known as

"Non-Topologically Extended for  $i$ -semi-inter open sets" (in short  $N.T.E.I.S.INTOs$ ).

**In Example (3.13),**  $(X, \tau)$  is  $T.E.I.S.INTOs$ , **but in Example (3.4),**  $(X, \tau)$  is  $N.T.E.I.S.INTOs$ .

**Remark.4.2.** Let  $X$  be a finite and non empty set and let  $K_i \subset X, \forall i = 1, 2, \dots, n$  satisfies the following conditions:

1.  $\emptyset, X \in \tau$ .
  2.  $\cup_{i=1}^n K_i \in \tau \quad \forall i = 1, 2, \dots, n$
  3.  $K_i \cap K_j = \emptyset \quad \forall i \neq j$  and  $K_1 \cup K_2 \cup \dots \cup K_n = X$
- If  $\tau = \{\emptyset, X\} \cup \{\cup K_i\}_{i=1}^n$  then  $(X, \tau^{ISINT})$  is a  $TS$  and  $(X, \tau)$  is  $T.E.I.S.INTOs$ .

**For Example.4.3.** Let  $X = \{p, q, u, v\}, \tau = \{\emptyset, \{p, q\}, \{u, v\}, X\}$

$$\tau^C = \{\emptyset, \{u, v\}, \{p, q\}, X\}$$

$$\tau^I = \{\emptyset, \{p, q\}, \{u, v\}, \{p\}, \{q\}, \{u\}, \{v\}, X\}$$

$$\tau^S = \{\emptyset, \{p, q\}, \{u, v\}, X\}$$

$$\tau^{SINT} = \{\emptyset, \{p, q\}, \{u, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, \{q, u, v\}, X\}$$

$$\tau^{ISINT} = \{\emptyset, \{p, q\}, \{u, v\}, X\}$$

It's clear that  $(X, \tau)$  is a  $TS$  and  $\tau = \tau^{ISINT}$  (Theorem.3.15.). Hence  $(X, \tau^{ISINT})$  is a  $TS$  Therefore  $(X, \tau)$  is  $T.E.I.S.INTOs$ .

**Remark.4.4.** Let  $X$  be a finite and non empty set and let  $K_i \subset X, \forall i = 1, 2, \dots, n$  be any partition of  $X$ . If  $\tau = \{\emptyset, X\} \cup \{\cup K_i\}_{i=1}^n$ , then in the following cases is not necessary that  $(X, \tau^{ISINT})$  be a  $TS$ . That means  $(X, \tau)$  is not necessarily to be  $T.E.I.S.INTOs$ .

Case 1:

1.  $\emptyset, X \in \tau$ .
2.  $\cup_{i=1}^n K_i \in \tau \quad \forall i = 1, 2, \dots, n$
3.  $K_i \cap K_j \neq \emptyset \quad \forall i \neq j \quad \cap_{i=1}^n K_i \in \tau, \quad \forall i = 1, 2, \dots, n$  and  $K_1 \cup K_2 \cup \dots \cup K_n \cup F = X$   
where  $F = (K_1 \cup K_2 \cup \dots \cup K_n)^c$ .

Case 2:

1.  $\emptyset, X \in \tau$ .
2.  $\cup_{i=1}^n K_i \in \tau \quad \forall i = 1, 2, \dots, n$
3.  $K_i \cap K_j \neq \emptyset \quad \forall i \neq j \quad \cap_{i=1}^n K_i \in \tau, \quad \forall i = 1, 2, \dots, n$  and  $K_1 \cup K_2 \cup \dots \cup K_n = X$

Case 3:

1.  $\emptyset, X \in \tau$ .
2.  $\cup_{i=1}^n K_i \in \tau \quad \forall i = 1, 2, \dots, n$
3.  $K_i \cap K_j = \emptyset \quad \forall i \neq j \quad \cap_{i=1}^n K_i \in \tau, \quad \forall i = 1, 2, \dots, n$  and  $K_1 \cup K_2 \cup \dots \cup K_n \cup F = X$

where  $F = (K_1 \cup K_2 \cup \dots \cup K_n)^c$ .

**Case 1. Example.4.5.** Let  $X = \{p, q, u, v\}$ ,  $\tau = \{\phi, \{p, q\}, \{u\}, \{p, q, u\}, X\}$ ,

**Where**  $F = (K_1 \cup K_2 \cup K_3)^c = \{v\}$

$\tau^C = \{\phi, \{u, v\}, \{p, q, v\}, \{v\}, X\}$

$\tau^I = \{\phi, \{p\}, \{q\}, \{u\}, \{p, q\}, \{p, v\}, \{p, q, u\}, \{p, u\}, \{p, q, v\}, \{p, u, v\}, \{q, u\}, \{q, v\}, \{u, v\},$

$\{q, u, v\}, X\}$

$\tau^S = \{\phi, \{u\}, \{p, q\}, \{p, q, v\}, \{u, v\}, \{p, q, u\}, X\}$

$\tau^{SINT} = \{\phi, \{u\}, \{p, q\}, \{p, u\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, \{q, u\}, \{u, v\}, \{q, u, v\}, X\}$

$\tau^{ISINT} = \{\phi, \{u\}, \{p, q\}, \{q, u\}, \{p, q, u\}, \{p, q, v\}, \{p, u\}, \{p, u, v\}, \{u, v\}, \{q, u, v\}, X\}$

$(X, \tau^{ISINT})$  is not *TS* because  $\{p, q\} \cap \{q, u\} = \{q\} \notin \tau^{ISINT}$ . Hence  $(X, \tau)$  is *N.T.E.I.S.INTOs*.

**Case 2. Example .4.6.** Let  $X = \{p, q, u, v\}$ ,  $\tau = \{\phi, \{p\}, \{p, q\}, \{u, v\}, \{p, u, v\}, X\}$

$\tau^C = \{\phi, \{q, u, v\}, \{u, v\}, \{p, q\}, \{q\}, X\}$

$\tau^I = \{\phi, \{p\}, \{q\}, \{u\}, \{v\}, \{p, q\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, \{u, v\}, X\}$

$\tau^S = \{\phi, \{p\}, \{p, q\}, \{u, v\}, \{p, u, v\}, X\}$

$\tau^{SINT} = \{\phi, \{p\}, \{p, q\}, \{u, v\}, \{p, u, v\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, X\}$

$\tau^{ISINT} = \{\phi, \{p\}, \{p, q\}, \{p, u\}, \{p, v\}, \{p, q, u\}, \{p, q, v\}, \{p, u, v\}, \{u, v\}, X\}$

$(X, \tau^{ISINT})$  is not *TS* because  $\{p, u\} \cap \{u, v\} = \{u\} \notin \tau^{ISINT}$ . Hence  $(X, \tau)$  is *N.T.E.I.S.INTOs*.

**Case 3. . In Example.3.4. .** Let  $X = \{p, q, u, v\}$ ,  $\tau = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, X\}$

where  $F = (K_1 \cup K_2 \cup K_3)^c = \{q\}$

$\tau^{ISINT} = \{\phi, \{p\}, \{u, v\}, \{p, u, v\}, \{p, q\}, \{p, u\}, \{p, q, u\}, \{p, v\}, \{p, q, v\}, \{q, u, v\}, X\}$  is not a *TS* because  $\{p, q\} \cap \{q, u, v\} = \{q\} \notin \tau^{ISINT}$ . Hence  $(X, \tau^{ISINT})$  is not *TS* and  $(X, \tau)$  is *N.T.E.I.S.INTOs*.

**Theorem.4.7.** Let  $X \neq \emptyset$  be a finite set,  $\tau = \{\emptyset, K, X\}$  where  $X = \{o_1, o_2, \dots, o_n\}$ , and  $K$  is a subset of  $X$  and containing one or two elements .....up to  $(n - 1)$  elements. Then  $(X, \tau^{ISINT})$  is a *TS*.(i.e.  $(X, \tau)$  is a *T.E.I.S.INTOs*).

Proof: It's clear that  $(X, \tau)$  is a *TS*.

Case (1): If  $K = \{o_1\}$ ,  $\tau = \{\emptyset, K, X\}$ . *Os* in  $(X, \tau)$  are:  $\emptyset, K, X$ . *Cs* are:  $X, \{o_2, o_3, \dots, o_n\}, \emptyset$ .

By definition of *IOS* [2,3]. We have:

$\tau^I = \{\emptyset, K, \{o_1, o_2\}, \{o_1, o_3\}, \dots, \{o_1, o_n\}, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\},$

$\dots, \{o_1, o_3, o_4, \dots, o_n\}, \{o_1, o_2, \dots, o_n\} = X\}$ .

By definition of *SOS* [7]. We have:

$\tau^S = \{\emptyset, K, \{o_1, o_2\}, \{o_1, o_3\}, \dots, \{o_1, o_n\}, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\},$

$\dots, \{o_1, o_3, o_4, \dots, o_n\}, \{o_1, o_2, \dots, o_n\} = X\}$ .

By definition of *S.INTOs* we have:

$\tau^{SINT} = \{\emptyset, K, \{o_1, o_2\}, \{o_1, o_3\}, \dots, \{o_1, o_n\}, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\},$

$\dots, \{o_1, o_3, o_4, \dots, o_n\}, \{o_1, o_2, \dots, o_n\} = X\}$ .

By definition of *I.S.INTOs* we have:

$\tau^{ISINT} = \{\emptyset, K, \{o_1, o_2\}, \{o_1, o_3\}, \dots, \{o_1, o_n\}, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\},$

$\dots, \{o_1, o_3, o_4, \dots, o_n\}, \{o_1, o_2, \dots, o_n\} = X\}$ .

We see that  $(X, \tau^{ISINT})$  is a *TS*. Hence  $(X, \tau)$  is *T.E.I.S.INTOs*.

Case (2): If  $K = \{o_1, o_2\}$ ,  $\tau = \{\emptyset, K, X\}$ . *Os* in  $(X, \tau)$  are:  $\emptyset, K, X$ . *Cs* are:  $X, \{o_3, o_4, \dots, o_n\}, \emptyset$ . By definition of *IOS* [2,3].

We have:  $\tau^I = \{\emptyset, K, \{o_1\}, \{o_2\}, \{o_1, o_3\}, \{o_1, o_4\}, \dots, \{o_1, o_n\}, \{o_2, o_3\}, \{o_2, o_4\}, \dots, \{o_2, o_n\},$

$\{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\}, \{o_1, o_2, o_3, o_4\}, \dots, \{o_1, o_2, \dots, o_n\} = X\}$ .

. By definition of *SOs* [7], we have:

$$\tau^S = \{\emptyset, K, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\}, \{o_1, o_2, o_3, o_4\}, \{o_1, o_2, o_3, o_4, o_5\}, \dots, \{o_1, o_2, o_3, o_n\}, \{o_1, o_2, \dots, o_n\} = X\}.$$

By definition of *S.INTOs* we have:

$$\tau^{SINT} = \{\emptyset, K, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \dots, \{o_1, o_2, o_n\}, \{o_1, o_2, o_3, o_4\}, \{o_1, o_2, o_3, o_4, o_5\}, \{o_1, o_2, o_3, o_n\}, \dots, \{o_1, o_2, \dots, o_n\} = X\}.$$

By definition of *I.S.INTOs* we have:

$$\tau^{ISINT} = \{\emptyset, K, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\}, \{o_1, o_2, o_3, o_4\}, \dots, \{o_1, o_2, o_3, \dots, o_n\} = X\}.$$

We see that  $(X, \tau^{ISINT})$  is a *TS*. Hence  $(X, \tau)$  is *T.E.I.S.INTOs*.

Case (3): Similarly, we can prove that if  $K = \{o_1, o_2, o_3\}$

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Case (n): If  $K = \{o_1, o_2, \dots, o_{n-1}\}$ ,  $\tau = \{\emptyset, K, X\}$ . *Os* in  $(X, \tau)$  are:  $\emptyset, K, X$ . *Cs* are:  $X, \{o_n\}, \emptyset$ . By definition of *IOs* [3]. We

$$\text{have: } \tau^I = \{\emptyset, K, \{o_1\}, \{o_2\}, \dots, \{o_{n-1}\}, \{o_1, o_2\}, \{o_1, o_3\}, \{o_1, o_4\} \dots \{o_1, o_n\}, \{o_2, o_3\}, \dots, \{o_2, o_4\}, \dots, \{o_2, o_n\}, \{o_1, o_2, o_3\}, \{o_1, o_2, o_4\} \dots, \{o_1, o_2, \dots, o_n\}, \{o_1, o_2, o_3, o_4\}, \dots, \{o_1, o_2, \dots, o_n\} = X\}.$$

By definition of *SOs* [7], we have:  $\tau^S = \{\emptyset, K, X\}$ .

By definition of *S.INTOs* we have:  $\tau^{SINT} = \{\emptyset, K, X\}$

By definition of *I.S.INTOs* we have:  $\tau^{ISINT} = \{\emptyset, K, X\}$ .

We see that  $(X, \tau^{ISINT})$  is a *TS*. Hence  $(X, \tau)$  is *T.E.I.S.INTOs*.

**Example.4.8.** Let  $X = \{p, q, u\}$ ,  $\tau = \{\emptyset, \{u\}, X\}$ ,  $\tau^{ISINT} = \{\emptyset, \{u\}, \{q, u\}, \{p, u\}, X\}$

By (Theorem.4.7) we have,  $(X, \tau^{ISINT})$  is a *TS*. Hence  $(X, \tau)$  is *T.E.I.S.INTOs*.

### 5. Some Properties of *i* –Semi-Inter- Open Sets.

Throughout this section,  $(X, \tau)$  is always *T.E.I.S.INTOs* (i.e  $(X, \tau^{ISINT})$  is always a *TS*), with some examples and theorems.

**Definition.5.1.** Let  $(X, \tau^{ISINT})$  be *TS*,  $K \subseteq X$ . An "*i* –Semi-Inter-limit point of *K*" is a point  $x \in X$  if, for any *i* –semi-inter-open set *S* containing  $x$ ,  $(K \cap S) \setminus \{x\} \neq \emptyset$ , the collection of all of *K*'s *i* –semi-inter-limit points is called "*i* –Semi-Inter-derived set of *K*" and denoted by  $D_{ISINT}(K)$ . *K* is *i* –semi-inter-closed set if and only if  $D_{ISINT}(K) \subseteq K$ .

**Example.5.2.** Let  $X = \{p, q, u\}$ ,  $\tau = \{\emptyset, \{p\}, X\}$

$$\tau^C = \{\emptyset, \{q, u\}, X\}$$

$$\tau^I = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$$

$$\tau^S = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$$

$$\tau^{SINT} = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$$

$$\tau^{ISINT} = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$$

Where  $(X, \tau^{ISINT})$  is a *TS*. Let  $K_1 = \{p, q\}, K_2 = \{q, u\}$ .

$$\tau^{CSINT} = \{\emptyset, \{q, u\}, \{u\}, \{q\}, X\}.$$

$p \in X, p \in \{p\}, \{p, q\}, \{p, u\}, X$ .  $K_1 \cap \{p\} - \{p\} = \emptyset$ . Hence  $p \notin D_{ISINT}(K_1)$ .

$q \in X, q \in \{p, q\}, X$ .  $K_1 \cap \{p, q\} - \{q\} \neq \emptyset, K_1 \cap X - \{q\} \neq \emptyset$ , Hence  $q \in D_{ISINT}(K_1)$ .

$u \in X, u \in \{p, u\}, X$ .  $K_1 \cap \{p, u\} - \{u\} \neq \emptyset, K_1 \cap X - \{u\} \neq \emptyset$ , Hence  $u \in D_{ISINT}(K_1)$ .

Therefore,  $D_{ISINT}(K_1) = \{q, u\} \not\subseteq K_1 \Rightarrow K_1$  is not *I.S.INTC*.

Similarly,  $D_{ISINT}(K_2) = \emptyset \subset K_2 \Rightarrow K_2$  is *I.S.INTC*.

**Definition.5.3.** Suppose  $(X, \tau^{ISINT})$  be *TS*,  $K \subseteq X$ . The intersection of all *i* –semi-inter closed sets comprising *K* is known as "*i* –Semi-Inter-closure of *K*", indicated by  $CL_{ISINT}(K)$ , and defined as:  $CL_{ISINT}(K) = \bigcap_{i \in \Lambda} F_i \cdot K \subseteq F_i, \forall F_i$ , where  $F_i$  is *i* –semi-inter closed set,  $\forall i$  in a *TS*  $(X, \tau^{ISINT})$ .  $CL_{ISINT}(K)$  is the smallest *i* –semi-inter closed set containing *K*.

**Definition.5.4.** Suppose  $(X, \tau^{ISINT})$  be a  $TS$ ,  $K \subseteq X$ . The union of all  $i$ -semi-inter *open* sets contained in  $K$  is known as " $i$  – Semi-Inter-Interior of  $K$ ", indicated by  $Int_{ISINT}(K)$ , and defined as:  $Int_{ISINT}(K) = \cup_{i \in \Lambda} S_i, S_i \subseteq K, \forall i$ . Where  $S_i$  is  $i$  – semi-inter- *open* set,  $\forall i$  in a  $TS (X, \tau^{ISINT})$ .  $Int_{ISINT}(K)$  is the largest  $i$  – semi-inter *open* set contained in  $K$ .

**Definition.5.5.** Suppose  $(X, \tau^{ISINT})$  be a  $TS$ ,  $K \subseteq X$ . The  $i$  – semi-inter-interior of the complement of  $K$  is called the " $i$  – Semi-Inter-Exterior of a set  $K$ ", and denoted by  $Ext_{ISINT}(K)$ , and defined as:  $Ext_{ISINT}(K) = Int_{ISINT}(K^c)$ .

**Definition.5.6.** Suppose  $(X, \tau^{ISINT})$  be a  $TS$ ,  $K \subseteq X$ . " $i$  – Semi-Inter-boundary of a set  $K$ " is denoted by  $b_{ISINT}(K)$ , and defined as:  $b_{ISINT}(K) = (Int_{ISINT}(K) \cup Ext_{ISINT}(K))^c$ .

**Theorem.5.7.** Let  $(X, \tau^{ISINT})$  be a  $TS$ , for  $K, M \subseteq X$ , the following propositions hold:

- i.  $D_{ISINT}(K) \subseteq D(K)$ . (Where  $D(K)$  is the derived of a set  $K$  [12].
- ii. If  $K \subseteq M$ , then  $D_{ISINT}(K) \subseteq D_{ISINT}(M)$ .
- iii.  $D_{ISINT}(K \cup M) = D_{ISINT}(K) \cup D_{ISINT}(M)$ .
- iv.  $D_{ISINT}(K \cap M) \subseteq D_{ISINT}(K) \cap D_{ISINT}(M)$ .
- v. If  $x \in D_{ISINT}(K)$ , then  $x \in D_{ISINT}(K \setminus \{x\})$ .

Proof:

i. It suffices to observe that every  $O_S$  is  $I.S.INTOs$ .

ii. Let  $x \in D_{ISINT}(K)$ , then for any  $I.S.INTOs, S$  containing  $x$ , we have:  $(K \cap S) \setminus \{x\} \neq \emptyset \dots(1)$ .

Since  $K \subseteq M \Rightarrow (K \cap S) \subseteq (M \cap S) \Rightarrow (K \cap S) \setminus \{x\} \subseteq (M \cap S) \setminus \{x\} \neq \emptyset$  (by(1))  $\Rightarrow (M \cap S) \setminus \{x\} \neq \emptyset \Rightarrow x \in D_{ISINT}(M)$ . Hence  $D_{ISINT}(K) \subseteq D_{ISINT}(M)$ .

iii. since  $K \subseteq K \cup M, M \subseteq K \cup M$ . By(ii) we have:

$$D_{ISINT}(K) \subseteq D_{ISINT}(K \cup M), D_{ISINT}(M) \subseteq D_{ISINT}(K \cup M) \\ \Rightarrow D_{ISINT}(K) \cup D_{ISINT}(M) \subseteq D_{ISINT}(K \cup M) \dots\dots(*)$$

Now let  $x \notin D_{ISINT}(K), x \notin D_{ISINT}(M)$ . Then there exist two  $I.S.INTOs. (S_x^A, S_x^B)$  containing  $x$  i.e.  $(K \cap S_x^A) \setminus \{x\} = \emptyset, (M \cap S_x^B) \setminus \{x\} = \emptyset$ . Let  $S = S_x^A \cap S_x^B$ . Where  $S$  is an  $I.S.INTOs$  (because  $(X, \tau^{ISINT})$  is a topological space).

$$(K \cup M) \cap S \setminus \{x\} = ((K \cap S) \cup (M \cap S)) \setminus \{x\} = ((K \cap S) \setminus \{x\}) \cup ((M \cap S) \setminus \{x\}) = \emptyset \cup \emptyset = \emptyset$$

Hence  $x \notin D_{ISINT}(K \cup M)$

$$\Rightarrow D_{ISINT}(K \cup M) \subseteq D_{ISINT}(K) \cup D_{ISINT}(M) \dots\dots(**)$$

By (\*) and (\*\*) we have:  $D_{ISINT}(K \cup M) = D_{ISINT}(K) \cup D_{ISINT}(M)$ .

iv. since  $K \cap M \subseteq K, K \cap M \subseteq M$ . By (ii), we have:  $D_{ISINT}(K \cap M) \subseteq D_{ISINT}(K), D_{ISINT}(K \cap M) \subseteq D_{ISINT}(M)$ .

Hence  $D_{ISINT}(K \cap M) \subseteq D_{ISINT}(K) \cap D_{ISINT}(M)$ .

v. Let  $x \in D_{ISINT}(K) \Rightarrow$  for any  $I.S.INTOs, S$  containing  $x$ .

$$\text{We have: } (K \cap S) \setminus \{x\} \neq \emptyset \Rightarrow (K \cap S) \cap \{x\}^c = (K \cap \{x\}^c) \cap (S \cap \{x\}^c) = (K \setminus \{x\}) \cap (S \setminus \{x\}) \\ = ((K \setminus \{x\}) \cap S) \setminus \{x\} \neq \emptyset \Rightarrow x \in D_{ISINT}(K \setminus \{x\}).$$

**Example.5.8.** Let  $X = \{p, q, u\}, \tau = \{\phi, \{p\}, \{q, u\}, X\}$

and let  $K_1 = \{q, u\}, K_2 = \{u\}, K_3 = \{p\}$

$$\tau^c = \{\phi, \{q, u\}, \{p\}, X\}$$

$$\tau^I = \{\phi, \{p\}, \{q\}, \{q, u\}, \{u\}, X\}$$

$$\tau^S = \{\phi, \{p\}, \{q, u\}, X\}$$

$$\tau^{SINT} = \{\phi, \{p\}, \{p, q\}, \{p, u\}, \{q, u\}, X\}$$

$$\tau^{ISINT} = \{\phi, \{p\}, \{q, u\}, X\}$$

$$\tau^{ISINTC} = \{X, \{q, u\}, \{p\}, \emptyset\}$$

It is clear that  $(X, \tau)$  is  $T.E.I.S.INT$  (i.e  $(X, \tau^{ISINT})$  is a  $TS$ ).

i.  $D(K_1) = \{q, u\}, D_{ISINT}(K_1) = \{q, u\}, \Rightarrow D_{ISINT}(K_1) = D(K_1)$ .

ii.  $D_{ISINT}(K_2) = \{q\}, K_2 \subseteq K_1 \Rightarrow D_{ISINT}(K_2) \subseteq D_{ISINT}(K_1)$ .

iii.  $D_{ISINT}(K_3) = \emptyset, D_{ISINT}(K_1) \cup D_{ISINT}(K_3) = \{q, u\} \cup \emptyset = \{q, u\},$

$K_1 \cup K_3 = \{q, u\} \cup \{p\} = X, D_{ISINT}(K_1 \cup K_3) = D_{ISINT}(X) = \{q, u\},$

Hence  $D_{ISINT}(K_1 \cup K_3) = D_{ISINT}(K_1) \cup D_{ISINT}(K_3)$ .

iv.  $K_1 \cap K_2 = \{q, u\} \cap \{u\} = \{u\}, D_{ISINT}(K_1 \cap K_2) = \{q\},$

$D_{ISINT}(K_1) \cap D_{ISINT}(K_2) = \{q, u\} \cap \{q\} = \{q\}$ .  
Hence  $D_{ISINT}(K_1 \cap K_2) \subseteq D_{ISINT}(K_1) \cap D_{ISINT}(K_2)$ .  
v.  $q \in D_{ISINT}(K_1)$ ,  $K_1 \setminus \{q\} = \{u\}$ ,  $D_{ISINT}(K_1 \setminus \{q\}) = D_{ISINT}(\{u\})$ ,  
 $q \in X$ ;  $q \in \{q, u\}$ ,  $X$ , Since  $(K_1 \setminus \{q\} \cap \{q, u\}) \setminus \{q\} \neq \emptyset$   
and  $(K_1 \setminus \{q\} \cap X) \setminus \{q\} \neq \emptyset \Rightarrow q \in D_{ISINT}(K_1 \setminus \{q\})$ .

**Theorem.5.9.** Let  $(X, \tau^{ISINT})$  be a TS, for  $K, M \subseteq X$ , the following propositions hold.

- i.  $CL_{ISINT}(X) = X, CL_{ISINT}(\emptyset) = \emptyset$ .
- ii.  $CL_{ISINT}(K)$  is I. S. INTCS.
- iii.  $K \subseteq CL_{ISINT}(K)$ .
- iv.  $K = CL_{ISINT}(K)$  if and only if  $K$  is I. S. INTCS.
- v.  $CL_{ISINT}(K)$  is the smallest I. S. INTCS containing  $K$ .
- vi.  $CL_{ISINT}(K) = CL_{ISINT}(CL_{ISINT}(K))$ .
- vii.  $CL_{ISINT}(K \cup M) = CL_{ISINT}(K) \cup CL_{ISINT}(M)$ .
- viii.  $CL_{ISINT}(K) = K \cup D_{ISINT}(K)$ .

Proof: viii. by (iii) we have:  $K \subseteq CL_{ISINT}(K) \dots(1)$

and by (Theorem.5.7(ii)) we have:  $D_{ISINT}(K) \subseteq D_{ISINT}(CL_{ISINT}(K)) \dots(2)$

since  $CL_{ISINT}(K)$  is I. S. INTCS.  $\Rightarrow D_{ISINT}(CL_{ISINT}(K)) \subseteq CL_{ISINT}(K) \dots\dots(3)$

by (2) and (3) we have:  $D_{ISINT}(K) \subseteq CL_{ISINT}(K) \dots\dots(4)$

by (1) and (4), we have:  $K \cup D_{ISINT}(K) \subseteq CL_{ISINT}(K)$

Otherwise, let  $x \in CL_{ISINT}(K)$ . If  $x \in K$ , then the proof is complete; if  $x \notin K$ , all I. S. INTOs  $S$  that contains  $x$  intersects  $K$  at a point different from  $x$ .

So  $x \in D_{ISINT}(K)$ . Hence  $CL_{ISINT}(K) \subseteq K \cup D_{ISINT}(K)$ , which completes the proof.

**(Application of Theorem.5.9.)**

**In Example.5.2.** Let  $X = \{p, q, u\}, \tau = \{\phi, \{p\}, X\}$

and let  $K = \{p, q\}, M = \{u\}, N = \{q\}$ ,

$$\tau^{ISINT} = \{\phi, \{p\}, \{p, q\}, \{p, u\}, X\}$$

$$\tau^{CISINT} = \{X, \{q, u\}, \{u\}, \{q\}, \emptyset\}$$

It is clear that  $(X, \tau)$  is T. E. I. S. INT (i.e  $(X, \tau^{ISINT})$  is a TS.

$$CL_{ISINT}(K) = X, CL_{ISINT}(M) = \{q, u\} \cap \{u\} \cap X = \{u\}$$

$$CL_{ISINT}(N) = \{q, u\} \cap \{q\} \cap X = \{q\}.$$

$$i. CL_{ISINT}(X) = X, CL_{ISINT}(\emptyset) = \emptyset.$$

$$ii. CL_{ISINT}(K), CL_{ISINT}(M), CL_{ISINT}(N) \text{ are I. S. INTCS.}$$

$$iii. K \subseteq CL_{ISINT}(K).$$

$$iv. M = \{u\} \text{ is a I. S. INTCS then } M = CL_{ISINT}(M).$$

$$vi. CL_{ISINT}(CL_{ISINT}(M)) = CL_{ISINT}(\{u\}) = \{q, u\} \cap \{u\} \cap X = \{u\},$$

$$i. e CL_{ISINT}(M) = CL_{ISINT}(CL_{ISINT}(M)).$$

$$vii. M \cup N = \{u\} \cup \{q\} = \{q, u\}$$

$$CL_{ISINT}(M \cup N) = \{q, u\} \cap X = \{q, u\},$$

$$CL_{ISINT}(M) \cup CL_{ISINT}(N) = \{u\} \cup \{q\} = \{q, u\},$$

i. e  $CL_{ISINT}(M \cup N) = CL_{ISINT}(M) \cup CL_{ISINT}(N)$ .

viii.  $D_{ISINT}(K) = \{q, u\}$ ,  $K \cup D_{ISINT}(K) = \{p, q\} \cup \{q, u\} = X$ , i.e.  $Cl_{ISINT}(K) = K \cup D_{ISINT}(K)$ .

**Theorem.5.10.** Let  $(X, \tau^{ISINT})$  be a TS, for  $K, M \subseteq X$ , the following propositions hold.

- i.  $Int_{ISINT}(K)$  is I.S. INTOs.
- ii.  $Int_{ISINT}(K) \subseteq K$ .
- iii.  $Int_{ISINT}(X) = X$ ,  $Int_{ISINT}(\emptyset) = \emptyset$ .
- iv.  $K = Int_{ISINT}(K)$  if and only if  $K$  is I.S. INTOs.
- v.  $Int_{ISINT}(K)$  is the largest I.S. INTOs contained in  $K$ .
- vi.  $Int_{ISINT}(Int_{ISINT}(K)) = Int_{ISINT}(K)$ .
- vii.  $Int_{ISINT}(K) = K \setminus D_{ISINT}(X \setminus K)$ .
- viii.  $X \setminus Int_{ISINT}(K) = CL_{ISINT}(X \setminus K)$
- ix. If  $K \subseteq M$ , then  $Int_{ISINT}(K) \subseteq Int_{ISINT}(M)$ .
- x.  $Int_{ISINT}(K) \cup Int_{ISINT}(M) \subseteq Int_{ISINT}(K \cup M)$ .
- xi.  $Int_{ISINT}(K \cap M) \subseteq Int_{ISINT}(K) \cap Int_{ISINT}(M)$ .

Proof: vii. Let  $x \in K \setminus D_{ISINT}(X \setminus K) \Rightarrow x \notin D_{ISINT}(X \setminus K)$ . Thus, there exists an I.S. INTOs,  $S$  that contains  $x$  so that  $S \cap (X \setminus K) = \emptyset \Rightarrow x \in S \subseteq K \Rightarrow x \in \bigcup_{i \in \Lambda} S_{ISINT} = Int_{ISINT}(K) \Rightarrow K \setminus D_{ISINT}(X \setminus K) \subseteq Int_{ISINT}(K)$

Otherwise, let  $x \in Int_{ISINT}(K) \Rightarrow x \notin D_{ISINT}(X \setminus K)$ .  
 Given that  $Int_{ISINT}(K)$  is I.S. INTOs and  $Int_{ISINT}(K) \cap (X \setminus K) = \emptyset$ .

Hence,  $Int_{ISINT}(K) = K \setminus D_{ISINT}(X \setminus K)$ .

viii.  $X \setminus Int_{ISINT}(K) = X \setminus (K \setminus D_{ISINT}(X \setminus K)) \dots \dots$  (by vii).  
 $= (X \setminus K) \cup D_{ISINT}(X \setminus K) = CL_{ISINT}(X \setminus K)$ .

ix. since  $K \subseteq M$  and  $Int_{ISINT}(K) \subseteq K$ ,  $Int_{ISINT}(M) \subseteq M$ .

Hence,  $Int_{ISINT}(K) \subseteq Int_{ISINT}(M)$ .

x. since  $K \subseteq (K \cup M)$ ,  $M \subseteq (K \cup M)$ .by (ix) we have:

$Int_{ISINT}(K) \subseteq Int_{ISINT}(K \cup M)$ ,  $Int_{ISINT}(M) \subseteq Int_{ISINT}(K \cup M)$ .

Hence,  $Int_{ISINT}(K) \cup Int_{ISINT}(M) \subseteq Int_{ISINT}(K \cup M)$ .

xi. since  $K \cap M \subseteq K$ ,  $K \cap M \subseteq M$ .by (ix) we have:

$Int_{ISINT}(K \cap M) \subseteq Int_{ISINT}(K)$ ,  $Int_{ISINT}(K \cap M) \subseteq Int_{ISINT}(M)$ .

Hence,  $Int_{ISINT}(K \cap M) \subseteq Int_{ISINT}(K) \cap Int_{ISINT}(M)$ .

(Application of Theorem.5.10.)

**In Example.5.2.** Let  $X = \{p, q, u\}$ ,  $\tau = \{\emptyset, \{p\}, X\}$

and let  $K = \{p, q\}$ ,  $M = \{u\}$ ,  $N = \{q\}$ ,

$\tau^{ISINT} = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$

$\tau^{CISINT} = \{X, \{q, u\}, \{u\}, \{q\}, \emptyset\}$

It is clear that  $(X, \tau)$  is T. E. I. S. INT (i.e  $(X, \tau^{ISINT})$  is a TS.

$Int_{ISINT}(K) = \{p\} \cup \{p, q\} = \{p, q\}$ ,  $Int_{ISINT}(M) = \emptyset$ ,  $Int_{ISINT}(N) = \emptyset$ .

i.  $Int_{ISINT}(K)$ ,  $Int_{ISINT}(M)$ ,  $Int_{ISINT}(N)$  are I.S. INTOs.

ii.  $Int_{ISINT}(K) \subseteq K$ .

iii.  $Int_{ISINT}(X) = X$ ,  $Int_{ISINT}(\emptyset) = \emptyset$ .

iv.  $K = Int_{ISINT}(K)$  Then,  $K$  is I.S. INTOs.

vi.  $Int_{ISINT}(Int_{ISINT}(K)) = Int_{ISINT}(\{p, q\}) = \{p\} \cup \{p, q\} = \{p, q\}$ .

i.e  $Int_{ISINT}(Int_{ISINT}(K)) = Int_{ISINT}(K)$ .

vii.  $X \setminus K = \{u\}$  ,  $D_{ISINT}(X \setminus K) = D_{ISINT}(\{u\}) = \emptyset$ .

$A \setminus D_{ISINT}(X \setminus K) = \{p, q\} \setminus \emptyset = \{p, q\}$ . i.e.  $Int_{ISINT}(K) = K \setminus D_{ISINT}(X \setminus K)$

viii.  $CL_{ISINT}(X \setminus K) = CL_{ISINT}(\{u\}) = \{q, u\} \cap \{u\} \cap X = \{u\}$ ,

$X \setminus Int_{ISINT}(K) = X \setminus \{p, q\} = \{u\}$ , i.e.  $X \setminus Int_{ISINT}(K) = CL_{ISINT}(X \setminus K)$ .

ix.  $N \subseteq K$  Then  $Int_{ISINT}(N) = \emptyset \subseteq Int_{ISINT}(K) = \{p, q\}$ , i.e.  $Int_{ISINT}(N) \subseteq Int_{ISINT}(K)$ .

x.  $K \cup N = \{p, q\} \cup \{q\} = \{p, q\}$ ,  $Int_{ISINT}(K \cup N) = \{p, q\}$ ,

$Int_{ISINT}(K) \cup Int_{ISINT}(N) = \{p, q\} \cup \emptyset = \{p, q\}$ , i.e.  $Int_{ISINT}(K) \cup Int_{ISINT}(N) \subseteq Int_{ISINT}(K \cup N)$

xi.  $K \cap N = \{p, q\} \cap \{q\} = \{q\}$ ,

$Int_{ISINT}(K) \cap Int_{ISINT}(N) = \{p, q\} \cap \emptyset = \emptyset$ ,

$Int_{ISINT}(K \cap N) = Int_{ISINT}(\{q\}) = \emptyset$ , i.e.  $Int_{ISINT}(K \cap N) \subseteq Int_{ISINT}(K) \cap Int_{ISINT}(N)$ .

**Theorem.5.11.** Let  $(X, \tau^{ISINT})$  be a TS, for  $K, M \subseteq X$ , the following propositions hold.

i.  $Ext(K) \subseteq Ext_{ISINT}(K)$ , i.e.  $Ext(K) = Int(K^c)[12]$ .

ii.  $Ext_{ISINT}(K)$  is I. S. INTOS .

iii.  $Ext_{ISINT}(K) = Int_{ISINT}(X \setminus K) = X \setminus CL_{ISINT}(K)$ .

iv.  $Ext_{ISINT}(Ext_{ISINT}(K)) = Int_{ISINT}(CL_{ISINT}(K))$ .

v. If  $K \subseteq M$  then  $Ext_{ISINT}(K) \supseteq Ext_{ISINT}(M)$  .

vi.  $Ext_{ISINT}(K \cup M) \subseteq Ext_{ISINT}(K) \cup Ext_{ISINT}(M)$ .

vii.  $Ext_{ISINT}(K \cap M) \supseteq Ext_{ISINT}(K) \cap Ext_{ISINT}(M)$ .

viii.  $Ext_{ISINT}(X) = \emptyset$ ,  $Ext_{ISINT}(\emptyset) = X$  .

ix.  $Ext_{ISINT}(K) = Ext_{ISINT}(X \setminus Ext_{ISINT}(K))$ .

x.  $Int_{ISINT}(K) \subseteq Ext_{ISINT}(Ext_{ISINT}(K))$ .

Proof: iv.  $Ext_{ISINT}(Ext_{ISINT}(K)) = Ext_{ISINT}(X \setminus CL_{ISINT}(K))$ (from iii)

$= Int_{ISINT}(X \setminus (X \setminus CL_{ISINT}(K))) = Int_{ISINT}(CL_{ISINT}(K))$ .

ix.  $Ext_{ISINT}(X \setminus Ext_{ISINT}(K)) = Ext_{ISINT}(X \setminus Int_{ISINT}(X \setminus K)) = Int_{ISINT}(X \setminus (X \setminus Int_{ISINT}(X \setminus K)))$

$= Int_{ISINT}(Int_{ISINT}(X \setminus K)) = Int_{ISINT}(X \setminus K) = Ext_{ISINT}(K)$ .

x.  $Int_{ISINT}(K) \subseteq Int_{ISINT}(CL_{ISINT}(K))$  ( $K \subseteq CL_{ISINT}(K)$ )

$= Int_{ISINT}(X \setminus Int_{ISINT}(X \setminus K)) = Int_{ISINT}(X \setminus Ext_{ISINT}(K)) = Ext_{ISINT}(Ext_{ISINT}(K))$ .

The proof is complete.

(Application of Theorem.5.11.)

**In Example.5.2.** Let  $X = \{p, q, u\}$ ,  $\tau = \{\emptyset, \{p\}, X\}$

and let  $K = \{p, q\}$ ,  $M = \{u\}$ ,  $N = \{q\}$ ,

$\tau^{ISINT} = \{\emptyset, \{p\}, \{p, q\}, \{p, u\}, X\}$

$\tau^{CISINT} = \{X, \{q, u\}, \{u\}, \{q\}, \emptyset\}$

It is clear that  $(X, \tau)$  is T. E. I. S. INT (i.e  $(X, \tau^{ISINT})$  is a TS.

$Ext_{ISINT}(K) = Int_{ISINT}(K^c) = Int_{ISINT}(\{u\}) = \emptyset$ ,

$$\text{Ext}_{\text{ISINT}}(M) = \text{Int}_{\text{ISINT}}(M^c) = \text{Int}_{\text{ISINT}}(\{p, q\}) = \{p, q\},$$

$$\text{Ext}_{\text{ISINT}}(N) = \text{Int}_{\text{ISINT}}(N^c) = \text{Int}_{\text{ISINT}}(\{p, u\}) = \{p, u\},$$

$$\text{i. Ext}(N) = \text{Int}(N^c) = \text{Int}(\{p, u\}) = \{p\}, \{p\} \subseteq \{p, u\}, \text{ i.e. } \text{Ext}(N) \subseteq \text{Ext}_{\text{ISINT}}(N)$$

ii.  $\text{Ext}_{\text{ISINT}}(K), \text{Ext}_{\text{ISINT}}(M), \text{Ext}_{\text{ISINT}}(N)$  are I. S. INTOs.

$$\text{iii. } X \setminus N = X \setminus \{q\} = \{p, u\}, \text{ Int}_{\text{ISINT}}(X \setminus N) = \text{Int}_{\text{ISINT}}(\{p, u\}) = \{p, u\},$$

$$X \setminus \text{CL}_{\text{ISINT}}(N) = X \setminus \{q\} = \{p, u\}, \text{ i.e. } \text{Ext}_{\text{ISINT}}(N) = \text{Int}_{\text{ISINT}}(X \setminus N) = X \setminus \text{CL}_{\text{ISINT}}(N).$$

$$\text{iv. } \text{Ext}_{\text{ISINT}}(\text{Ext}_{\text{ISINT}}(N)) = \text{Ext}_{\text{ISINT}}(\{p, u\}) = \text{Int}_{\text{ISINT}}(\{q\}) = \emptyset,$$

$$\text{Int}_{\text{ISINT}}(\text{CL}_{\text{ISINT}}(N)) = \text{Int}_{\text{ISINT}}(\{q\}) = \emptyset, \text{ i.e. } \text{Ext}_{\text{ISINT}}(\text{Ext}_{\text{ISINT}}(N)) = \text{Int}_{\text{ISINT}}(\text{CL}_{\text{ISINT}}(N)).$$

$$\text{v. } N \subseteq K, \text{ Then } \text{Ext}_{\text{ISINT}}(N) \supseteq \text{Ext}_{\text{ISINT}}(K).$$

$$\text{vi. } K \cup N = \{p, q\} \cup \{q\} = \{p, q\},$$

$$\text{Ext}_{\text{ISINT}}(K \cup N) = \text{Int}_{\text{ISINT}}(\{p, q\})^c = \text{Int}_{\text{ISINT}}(\{u\}) = \emptyset,$$

$$\text{Ext}_{\text{ISINT}}(K) \cup \text{Ext}_{\text{ISINT}}(N) = \emptyset \cup \{p, u\} = \{p, u\}, \quad \emptyset \subseteq \{p, u\},$$

$$\text{i.e. } \text{Ext}_{\text{ISINT}}(K \cup N) \subseteq \text{Ext}_{\text{ISINT}}(K) \cup \text{Ext}_{\text{ISINT}}(N).$$

$$\text{vii. } K \cap N = \{q\}, \text{ Ext}_{\text{ISINT}}(K \cap N) = \text{Int}_{\text{ISINT}}(\{q\})^c = \{p, u\},$$

$$\text{Ext}_{\text{ISINT}}(K) \cap \text{Ext}_{\text{ISINT}}(N) = \emptyset \cap \{p, u\} = \emptyset, \quad \emptyset \subseteq \{p, u\},$$

$$\text{i.e. } \text{Ext}_{\text{ISINT}}(K \cap N) \supseteq \text{Ext}_{\text{ISINT}}(K) \cap \text{Ext}_{\text{ISINT}}(N).$$

$$\text{viii. } \text{Ext}_{\text{ISINT}}(X) = \text{Int}_{\text{ISINT}}(X)^c = \text{Int}_{\text{ISINT}}(\emptyset) = \emptyset,$$

$$\text{Ext}_{\text{ISINT}}(\emptyset) = \text{Int}_{\text{ISINT}}(\emptyset)^c = \text{Int}_{\text{ISINT}}(X) = X.$$

$$\text{ix. } X \setminus \text{Ext}_{\text{ISINT}}(N) = X \setminus \{p, u\} = \{q\},$$

$$\text{Ext}_{\text{ISINT}}(X \setminus \text{Ext}_{\text{ISINT}}(N)) = \text{Ext}_{\text{ISINT}}(\{q\}) = \text{Int}_{\text{ISINT}}(\{q\})^c = \text{Int}_{\text{ISINT}}(\{p, u\}) = \{p, u\},$$

$$\text{i.e. } \text{Ext}_{\text{ISINT}}(N) = \text{Ext}_{\text{ISINT}}(X \setminus \text{Ext}_{\text{ISINT}}(N)).$$

$$\text{x. } \text{Ext}_{\text{ISINT}}(\text{Ext}_{\text{ISINT}}(K)) = \text{Ext}_{\text{ISINT}}(\emptyset) = X,$$

$$\text{Int}_{\text{ISINT}}(K) = \{p, q\}, \{p, q\} \subseteq X, \text{ i.e. } \text{Int}_{\text{ISINT}}(K) \subseteq \text{Ext}_{\text{ISINT}}(\text{Ext}_{\text{ISINT}}(K)).$$

## 6. Conclusions

According to the above we concluded that in any  $(X, \tau)$  each " $\alpha$ -open set" is  $i$ -semi-inter open set, each "semi-open set" is semi-inter open set, and it is  $i$ -semi-inter open set, each  $\alpha$ -open set is semi-inter open set, each  $i$ -semi-inter open set is  $i$ -open set, and it is semi-inter open set, each open set is semi-inter open set and it is  $i$ -semi-inter open set. But in general, the converses are not necessary to be true. Also, we concluded that the family of  $i$ -semi-inter open sets does not necessarily need to be  $TS$ , and the intersection of any two  $i$ -semi-inter open sets is not necessarily be  $i$ -semi-inter open set.

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## 8. Declarations

**8.1 Ethics approval and consent to participate:** Not applicable.

**8.2 Consent for publication:** Not applicable.

**8.3 Availability of Data and Materials:** Data will be provided upon receiving a valid request.

**8.4 Conflicts of interest:** The authors declare that there is no conflict of interest

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## المجاميع شبه المفتوحة الداخلية من النمط $i$ – في الفضاءات التوبولوجية

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### المستخلص:

يهدف هذا البحث إلى تقديم صنفين جديدين من المجاميع شبه المفتوحة، أطلقنا عليه اسم المجاميع شبه المفتوحة الداخلية من النمط  $i$  والمجاميع شبه المفتوحة الداخلية في الفضاءات التوبولوجية. وتم التحقق من خصائص هذه المجاميع وعلاقتها بأصناف أخرى من المجاميع من خلال مجموعة من البراهين الرياضية، فعلى سبيل المثال برهنا انه في اي فضاء توبولوجي  $(X, \tau)$ : كل مجموعة مفتوحة من النمط  $\alpha$  تكون مجموعة شبه مفتوحة داخلية من النمط  $i$  وشبه مفتوحة داخلية, كل مجموعة شبه مفتوحة تكون مجموعة شبه مفتوحة داخلية وشبه مفتوحة داخلية من النمط  $i$ , كل مجموعة شبه مفتوحة داخلية من النمط  $i$  تكون مجموعة مفتوحة من النمط  $i$  وشبه مفتوحة داخلية, كل مجموعة مفتوحة تكون مجموعة شبه مفتوحة داخلية وشبه مفتوحة داخلية من النمط  $i$ . بشكل عام العكس غير صحيح. كذلك عرفنا صفة الموسعة وغير الموسعة توبولوجيا للمجاميع شبه المفتوحة الداخلية من النمط  $i$  حيث بينا بان عائلة المجاميع شبه المفتوحة الداخلية من النمط  $i$  ليس من الضروري ان تكون فضاءا توبولوجيا. مع بعض الخصائص والمميزات لهذه المجاميع مثل نقاط الغاية شبه المفتوحة الداخلية من النمط  $i$  وجبهة المجموعة شبه المفتوحة الداخلية من النمط  $i$  لأية مجموعة.