

Some Classes in Ideal Topological Spaces

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Abstract

This paper introduces new classes of open sets defined in ideal topological space. These classes, namely: $i-I$ -open, weakly $i-I$ -open, $ii-I$ -open, and weakly $ii-I$ -open. In addition to that, the current study gives new concepts of continuity of mapping between ideal topological spaces using these classes. It compares them with other classes of nearly open sets in ideal topological spaces. We prove that all open sets $\alpha-I$ -open, $semi-I$ -open, $ii-I$ -open, $weakly semi-I$ -open and weakly $ii-I$ -open sets are weakly $i-I$ -open for any ideal topological space. Additionally, we show that all $\alpha-I$ -continuous, $semi-I$ -continuous, and $ii-I$ -continuous mappings are $i-I$ -continuous. Finally, for ideal topological space $(\mathcal{M}, \mathcal{L}, I)$ and $D \subset \mathcal{M}$ satisfying $Int(D)^{\#} = Int(D)$, we show that the following statements are equal.:

- 1) D is open
- 2) D is $i-I$ -open and $D \cap H = Int(D)$ for some $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$
- 3) D is $semi-I$ -open

Similarly, we show that the following statements are equal.:

- 1) D is a closed set
- 2) $(D \cap F) = cl(D)$ for some $F \in \mathcal{L}^c$
- 3) D is $semi-I$ -closed

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Introduction

Kuratowski [6] and Vaidyanathaswamy [9] were the first to study the notion of ideal topological spaces. Jankovic and Hamlett [5] investigated further characteristics of perfect topological spaces in 1990. The classes i -open and ii -open sets which introduced recently by Mohammed –Askandar and Abdullah –Mohammed in [2] and [1], respectively. Given the ideal topological space $(\mathcal{M}, \mathcal{L}, I)$, we define $i-I$ -open and $ii-I$ -open sets via i -open and ii -open. The most important result is that for ideal Topological spaces, denoted by $(\mathcal{M}, \mathcal{L}, I)$ all $\alpha-I$ -open sets are $i-I$ -open and $ii-I$ -open. Further, all $semi-I$ -open sets are $i-I$ -open and $ii-I$ -open, and again, we prove that for ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ all $i-I$ -open set are weakly $i-I$ -open and $ii-I$ -open set is weakly $ii-I$ -open. With a discussion of $i-I$ -continuity and weakly $i-I$ -continuity.

Definition 1.1. [6]

An ideal I is a nonempty set of a topological space $(\mathcal{M}, \mathcal{L})$ is a collection of a subset of $\mathcal{M}$ which fulfills

- 1) $D \in I$ and $B \subseteq D$ implies $B \in I$,
- 2) $D \in I$ and $B \in I$ implies $D \cup B \in I$.

Definition 1.2.

Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and let $m \in \mathcal{M}$. Then a local function is a mapping from a family of a subset of $\mathcal{M}$ into itself, denoted by $(D)^\#(I, \mathcal{L}) = \{m \in \mathcal{M} : U \cap D \notin I \text{ for each } U \in \mathcal{L}(m)\}$ where $D \subseteq \mathcal{M}$, and $\mathcal{L}(m) = \{U \in \mathcal{L} : m \in U\}$. A Kuratowski closure operator $cl^\#$ is defined by $cl^\#(D) = D \cup D^\#(I, \mathcal{L})$. Note that we will write $D^\#$ for $D^\#(I, \mathcal{L})$.

Definition 1.3.

A subset D of Topological spaces $(\mathcal{M}, \mathcal{L})$ is called to be

a) α – open [8] if $D \subset Int(cl(Int(D)))$

b) semi – open [7] is $D \subset cl(Int(D))$

For a subset D of I.T.S. $(\mathcal{M}, \mathcal{L}, I)$ is called to be

c) α – I – open [4] if $D \subset Int(cl^\#(Int(D)))$

d) semi – I – open [4] is $D \subset cl^\#(Int(D))$

Definition 1.4.

A subset D of ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ is **i – I – open** if there exists $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$ such that $D \subset cl^\#(D \cap H)$.

The complement of **i – I – open set** is **i – I – closed**.

DEFINITION 1.5. A subset D of ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ is **ii – I – open** if there exist $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$ such that D is **i – I – open** and $Int(D) = H$. The complement of **ii – I – open** is **ii – I – closed**.

Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces Then

Theorem 1.1. Every open set is **i – I – open**.

Proof. For any open set H , we have $H \subset cl^\#(H \cap H) = cl^\#(H)$.

This implies that H is **i – I – open**. ■

Theorem 1.2. Let $(\mathcal{M}, \mathcal{L}, I)$ ideal Topological spaces If $D \subset H \in \mathcal{L}$. Then D is **i – I – open**.

Proof. Let $D \subset H \in \mathcal{L}$. $\rightarrow D \cap H = D$. Then $D \subset cl^\#(D \cap H)$. Therefore D is **i – I – open**. ■

Theorem 1.3. Each open set is **ii – I – open**.

Proof. For any open set H , we have $H \subset cl^\#(H \cap H) = cl^\#(H)$, and $Int(H) = H$. This means that H is **ii – I – open**.

Theorem 1.4. Every α – I – open set is **ii – I – open**.

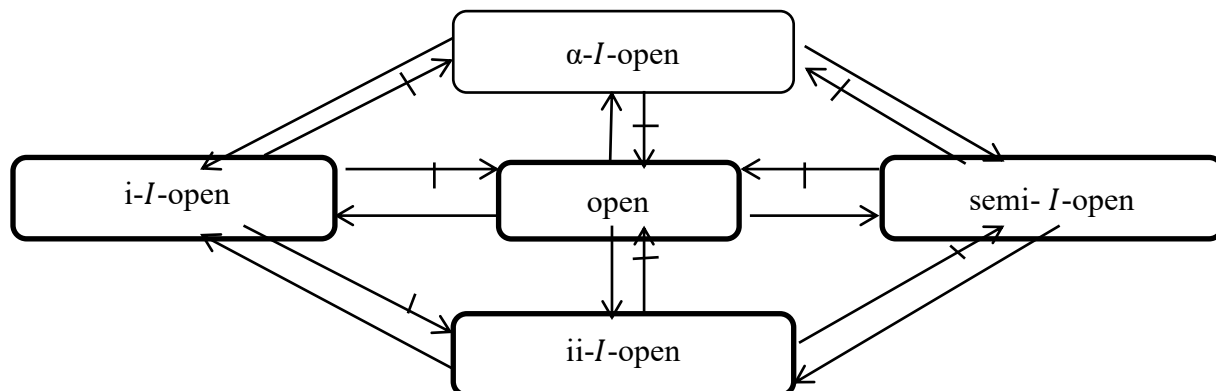
Proof. Let $D \subset \mathcal{M}$ be α – I – open. We have $D \subset Int(cl^\#(Int(D)))$. This implies that $D \subset cl^\#(Int(D))$. Now if

$Int(D) = \emptyset$. There is nothing to prove, so $Int(D) \neq \emptyset$. Therefore, put $Int(D) = H$ for some $H \in \mathcal{L}$. Hence $D \subset cl^\#(Int(D)) \subset cl^\#(D \cap H)$. That is D **ii – I – open**. ■

Theorem 1.5. Each **semi – I – open** set is **ii – I – open**

Proof. Direct. ■

From the above, we get the following Diagram:



2. Weakly Semi-I-open, Weakly i-I-open and Weakly ii-I-open

Definition 2.1.[3]

A subset D of I.T.S. $(\mathcal{M}, \mathcal{L}, I)$ is said to be a weakly **semi – I – open** set if $D \subset \text{cl}^\#(\text{Int}(\text{cl}(D)))$.

Remark 2.1.[3] Each semi-I-open set is weakly semi-I-open, but the converse is not true in general as shown in the following example

Example 1 Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a, b\}\}$ and $I = \{\emptyset, \{c\}\}$. Then $D = \{a\}$ is weakly **mi – I – open**, but not **semi – I – open**.

DEFINITION 2.2. Let $D \in \mathcal{M}$ of I.T.S. $(\mathcal{M}, \mathcal{L}, I)$ be said to be **weakly i – I – open** if there exists an open set $H \neq \{\mathcal{M}, \emptyset\}$ such that $H \subset \text{cl}^\#(\text{cl}(D \cap H))$.

DEFINITION 2.3. Let $D \in \mathcal{M}$ of I.T.S. $(\mathcal{M}, \mathcal{L}, I)$ be said to be **weakly ii – I – open** if there exists an open set $H \neq \{\mathcal{M}, \emptyset\}$ where D is weakly – i – I – open and $\text{Int}(D) = H$.

Theorem 2.1. Let $(\mathcal{M}, \mathcal{L}, I)$ ideal Topological spaces Then every **open** set is **akly i – I – open**, but the converse is not true.

Proof. Let $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$, we have $H \subset \text{cl}^\#((H \cap H)) \subset \text{cl}^\#(\text{cl}(H))$. This means H is **weakly i – I – open**. ■

Theorem 2.2. Let $(\mathcal{M}, \mathcal{L}, I)$ ideal Topological spaces Then every **i – I – open** set is **akly i – I – open**, but the converse is not true

Proof. let $D \subset \mathcal{M}$ be **i – I – open** set. For some $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$, we have $D \subset \text{cl}^\#((D \cap H)) \subset \text{cl}^\#(\text{cl}(D \cap H))$. This means D is **weakly i – I – open**. ■

Example 2

Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{a, b\}\}$ and $I = \{\emptyset, \{b\}\}$. Then $D = \{b, c\}$ is **weakly i – I – open**, however, not **i – I – open**.

Theorem 2.3. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces Then every **ii – I – open** set is **weakly ii – I – open**, but not conversely.

Proof. Direct. ■

Example 3

$\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{b\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}, \{c\}, \{a, c\}\}$. Then $D = \{a, c\}$ is **weakly ii – I – open**, but is not **ii – I – open**.

Corollary 2.1. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces Then every **weakly ii – I – open** set is **weakly i – I – open**, but not conversely.

Proof. Direct. ■

Corollary 2.2. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces Then each **open** set is **weakly ii – I – open** & **i – I – open**, but not conversely.

Proof. Direct. ■

DEFINITION 2.4. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. A map $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ is to be **i – I – continuous** if $f^{-1}(V)$ is **i – I – open** in $(\mathcal{M}, \mathcal{L}, I)$ for every open set V of (Y, σ) .

Theorem 2.4. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. Then every continuous mapping from $(\mathcal{M}, \mathcal{L}, I)$ to (Y, σ) is **i – I – continuous** but the converse is not true.

Proof. Let $U \in \sigma$. Since f is continuous mapping, then $f^{-1}(V)$ is an open set in $\mathcal{M}$, by Theorem 1.1. $f^{-1}(V)$ is an **i – I – open** in $\mathcal{M}$. Hence f is **i – I – continuous**. ■

Example 4

Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$. Then $Y = \{1, 2, 3\}$, $\sigma = \{Y, \emptyset, \{1\}, \{2\}, \{1, 2\}\}$ And Let $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ such that $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Then, f is **i – I – continuous**, but f is not **continuous** mapping.

DEFINITION 2.5. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. Then A map $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ is to be **weakly i – I – continuous** if $f^{-1}(V)$ is **weakly i – I – open** in $(\mathcal{M}, \mathcal{L}, I)$ for every open set V of (Y, σ) .

Theorem 2.5. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. Then every continuous mapping from $(\mathcal{M}, \mathcal{L}, I)$ to (Y, σ) is **weakly i – I – continuous** but the converse is not true.

Proof. Let $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ a continuous mapping and let V an open set in Y . Since f continuous, then $f^{-1}(V)$ is an open set in $\mathcal{M}$ from theorem 1.1 and theorem 2.1 f is weakly $i - I$ -continuous. ■

Example 5

Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a, b\}\}$ and $I = \{\emptyset, \{a\}\}$, $Y = \{1, 2, 3\}$, $\sigma = \{Y, \emptyset, \{1\}, \{1, 2\}\}$, $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$, such that $f(a) = 1$, $f(b) = 2$, $f(c) = 3$, $f^{-1}(1) = \{a\} \notin \mathcal{L}$. Then f is not continuous, but $\{a\} \cap \{a, b\} = \{a\}$ and $cl(\{a\}) = \mathcal{M}$, $\{a\} \subset cl^*(cl(\{a\} \cap \{a, b\}))$ and $f^{-1}(\{1, 2\}) = \{a, b\} \in \mathcal{L}$. Then f is weakly $i - I$ -continuous, but f is not continuous mapping.

Theorem 2.6. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ is $i - I$ -continuous. Then f is weakly $i - I$ -continuous, but the converse is not true.

Proof. Let $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ be an $i - I$ -continuous mapping and let V be an open set in Y . Since f is $i - I$ -continuous, then $f^{-1}(V)$ is $i - I$ -open in $\mathcal{M}$. From theorem 2.1 $f^{-1}(V)$ weakly $i - I$ -open. So f is weakly $i - I$ -continuous. ■

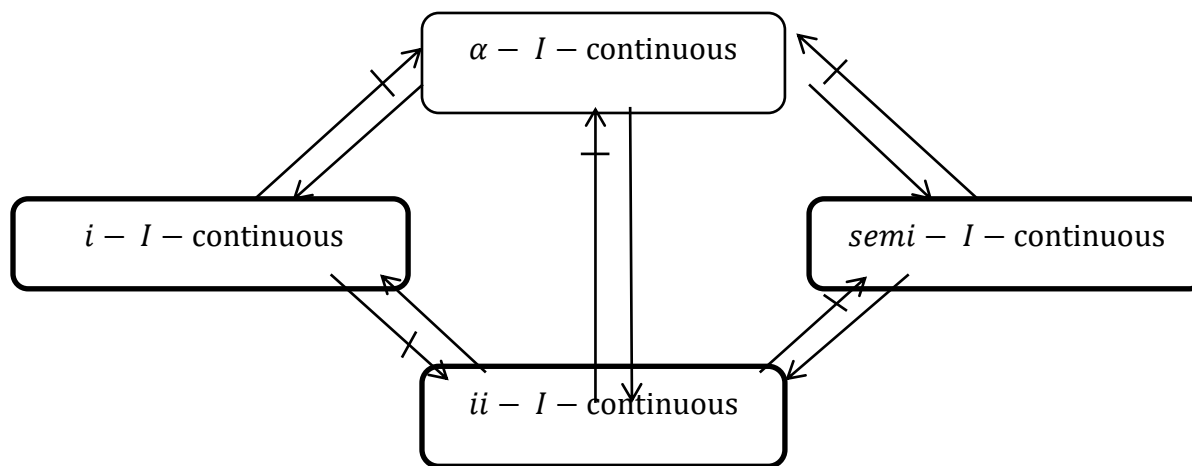
Example 6

Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}\}$ and $I = \{\emptyset, \{a\}\}$. $Y = \{1, 2, 3\}$, $\sigma = \{Y, \emptyset, \{1\}, \{1, 2\}\}$ σ , $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$, such that $f(a) = 1$, $f(b) = 2$, $f(c) = 3$. Then $f^{-1}(1) = \{a\} \in \mathcal{L}$ and $f^{-1}(\{1, 2\}) = \{a, b\} \notin \mathcal{L}$, $\{a\} \cap \{a, b\} = \{a\}$, then $cl^*(\{a\}) = \{a\} \cup \{a\}^* = \{a\}$ (since $\{a\}^* = \emptyset$). Therefore $\{a, b\} \notin cl^*(\{a\} \cap \{a, b\})$, but $cl(\{a\}) = X$, then $\{a, b\} \subset cl^*(cl(\{a\} \cap \{a, b\}))$. Then f is weakly $i - I$ -continuous, but f is not $i - I$ -continuous.

Theorem 2.7. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ is $\alpha - I$ -continuous. Then f is semi- I -continuous, $ii - I$ -continuous, and $i - I$ -continuous.

Proof. Let $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ be $\alpha - I$ -continuous. Then $f^{-1}(V)$ is $\alpha - I$ -open for every open set V of (Y, σ) . This mean $f^{-1}(V)$ is $mi - I$ -open. Then f is semi- I -continuous. By theorem 1.4. we have $f^{-1}(V)$ is $ii - I$ -open set. Therefore f is $ii - I$ -continuous. Thus f is $i - I$ -continuous. ■

From the previous theorem, we have



Definition 2.6.

A subset $D \subset \mathcal{M}$ is called **# – Int – perfect** if $(Int(D))^{\#} = Int(D)$

Theorem 2.8. Let $D \subset \mathcal{M}$ of ideal Topological spaces $(\mathcal{M}, \mathcal{L}, I)$ and D be **# – Int – perfect**. Then, the latter statement is equivalent.

1) D is open

2) D is **i – I – open** and $D \cap H = Int(D)$ for some $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$

3) D is **semi – I – open**

Proof. $[1 \Rightarrow 2]$. Direct

$[2 \Rightarrow 3]$

Let D be **i – I – open** and $D \cap H = Int(D)$. We have $D \subset cl^{\#}(D \cap H) = cl^{\#}(Int(D))$. Therefore D is **semi – I – open set**.

$[3 \Rightarrow 1]$

Let D be **semi – I – open**. We have $D \subset cl^{\#}(Int(D)) = Int(D) \cup (Int(D))^{\#} = Int(D) \cup Int(D) = Int(D)$. Therefore D is an open set. ■

Example 7

Let $\mathcal{M} = \{a, b, c\}$, $\mathcal{L} = \{\mathcal{M}, \emptyset, \{a\}\}$ and $I = \{\emptyset, \{a\}\}$.

$D = \{a\}$ is an open set. Then D is **i – I – open** and $D \cap D = D = Int(D)$. Therefore D is **semi – I – open**.

Theorem 2.9. Let $(\mathcal{M}, \mathcal{L}, I)$ be ideal Topological spaces and (Y, σ) be T.S. let $f: (\mathcal{M}, \mathcal{L}, I) \rightarrow (Y, \sigma)$ and D are **# – Int – perfect**, the following conditions are equivalent.

1) f is **continuous**

2) f is **i – I – continuous** and f is satisfy $f^{-1}(V) \cap H = Int(f^{-1}(V))$ for some $H \in \mathcal{L} \setminus \{\mathcal{M}, \emptyset\}$

3) f is **semi – I – continuous**

Proof. $[1 \Rightarrow 2]$. Clear

$[2 \Rightarrow 3]$

Let $V \subset Y$ since f is **i – I – continuous** it follows that $f^{-1}(V)$ is **i – I – open**, $f^{-1}(V) \subset cl^{\#}(f^{-1}(V) \cap H) = cl^{\#}(Int(f^{-1}(V)))$ then $f^{-1}(V)$ is **semi – I – open**. Therefore f is **semi – I – continuous**.

$[3 \Rightarrow 1]$

Let $U \in \sigma$. Since f is **semi – I – continuous**, it follows that $f^{-1}(V)$ is **semi – I – open**, so, $f^{-1}(V) \subset cl^{\#}(Int(f^{-1}(V))) = Int(f^{-1}(V))$. Then $f^{-1}(V)$ is open. Therefore f is **continuous**. ■

Theorem 2.10. For a subset D of space $(\mathcal{M}, \mathcal{L}, I)$ and $(Int(D))^{\#} = Int(D)$, the following conditions are equivalent.

1) D is a closed set

2) $(D \cap F) = cl(D)$ for some $F \in \mathcal{L}^c$

3) D is **semi – I – closed**

Proof. $[1 \Rightarrow 2]$. Clear

$[2 \Rightarrow 3]$ $cl^{\#}(Int(D)) = Int(D) \subset D$. Thus D is **semi – I – closed**

$[3 \Rightarrow 1]$

D is **semi – I – closed**. Then D^c is **semi – I – open**. We have $D^c \subset cl^{\#}(Int(D^c)) = Int(D^c)$. Therefore D^c is an open set. It follows D is closed. ■

Conclusions

This work concludes that each **open** set is **α – I – open**, **semi – I – open**, **ii – I – open**, and **i – I – open** set. Furthermore, each **α – I – continuous** mapping is **semi – I – continuous**, **ii – I – continuous**, and **i – I – continuous** mapping.

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Conflict of interest

The author has no conflict of interest.

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بعض الاصناف في الفضاءات التبولوجية المثالية

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الخلاصة

في هذه الدراسة , قدمنا اصناف جديدة من المجاميع المفتوحة في الفضاء التبولوجي المثالي تحديدا : مفتوحة من النمط $i - I$, مفتوحة من النمط (الضعيفة) $i - I$, مفتوحة من النمط $ii - I$ و مفتوحة من النمط (الضعيفة) $ii - I$. وكذلك , اعطينا مصطلحات جديدة لاستمرارية التطبيق بين الفضاءات التبولوجية المثالية باستخدام تلك الاصناف ونقارن مع اصناف اخرى من المجموعات الشبه المفتوحة في الفضاءات التبولوجية المثالية. اثبتنا ان كل مجموعة مفتوحة من النمط $I\alpha -$, (ضعيفة) مفتوحة من النمط شبه $I -$ و (ضعيفة) مفتوحة من النمط $ii - I$ هي (ضعيفة) مفتوحة من النمط $i - I$ لأي فضاء تبولوجي مثالي. اضافة الى ذلك اثبتنا ان كل التطبيقات المستمرة من النمط $I\alpha -$, المستمرة من النمط شبه $I -$ و المستمرة من النمط $ii - I$ بين الفضاءات التبولوجية المثالية هي مستمرة من النمط $i - I$. اخيرا , لأي فضاء تبولوجي مثالي $(M, L, I) \subset MD$ تحقق $Int(D)^{\#} = Int(D)$. اثبتنا ان الجمل الاتية تكون متكافئة :

(1) D هي مفتوحة (2) D هي مفتوحة من النمط $i - I$ و $Int(D) \cap D = Int(D)$ لبعض $H \in \mathcal{L} \setminus \{\emptyset, M\}$ (3) D هي مفتوحة من النمط شبه $I -$ على نفس المنوال اثبتنا ان الجمل الاتية تكون متكافئة :
(1) D هي مغلقة (2) D هي مغلقة من النمط $i - I$ و $cl(D) \cap D = Int(D)$ لبعض $F \in \mathcal{L}^c$ (3) D هي مغلقة من النمط شبه $I -$.