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Model Selection for the Mean Function of Kriging Models

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Abstract

Kriging models are used in many scientific disciplines to investigate the behavior of physical systems. In the Kriging model (KM), the response of the computer simulation code (CSC) is considered to have a Gaussian process (GP). To discover variables influencing responses, choosing a selection of variables or creating a strongly reduced regression model is a crucial process. Selecting some variables can prevent over-fitting or under-fitting in the predictions of data in KM. There have been just a few studies on the variable selection in KM. In this work, we suggest performing variable selection to construct a good model among the KM. The results of the proposed model selection are compared in terms of prediction accuracy with other models based on different forms of the mean function. The comparison is achieved by several measures that investigate the behavior of the KMs. Based on the results, the performance of the Forward selection and Backward selection based on AIC is the best. We apply KMs to several examples of computer simulation codes.

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1. Introduction

Researchers can now use sophisticated computer simulation codes (CSCs) to substitute physical experiments for advancements in computer technology. Additionally, inputs to CSCs frequently have high dimensions. Therefore, it may not be suitable to utilize a CSC for analysing the experiments, as it requires running numerous CSCs to optimize objective functions. In these situations, CSCs may be computationally expensive. Instead of using the simulation code, we can utilize a statistical model such as a Kriging model (KM) to roughly represent a relationship between a CSC's inputs and response values.

A CSC is typically deterministic or stochastic, which has a modest error. A KM was proposed as a metamodel for the CSC by [1]. KMs have been utilized to analyse data from computer experiments in several scientific fields. For example, in engineering, [2] and [3], in climate science, [4], in computer experiments, [5] , [6] and [7], in multivariate CSC, [8], and [9]. Numerous input variables may be present in many of these scenarios, and one must eliminate some of these variables to concentrate on the most crucial ones. Such a decrease may require additional analysis and can significantly lower the processing demands [10]. Finding which input variables have a significant impact on the system under study can also be done by ranking each one's relative relevance [11] . Variable selection could be beneficial in three important features: enhancing the predictors' efficacy, offering more time- and cost-efficient predictors, and giving users a clearer grasp of the mechanisms that underlie data generation. KM can be approximated using the same logic and methodology. However, very little study has been done on the variable selection in KM [10]; [12], and the majority of earlier studies used a straightforward KM without variable or parameter selection. In this study, we perform variable selection using forward selection and backward selection, and we apply the KMs to a test function and a real example.

In keeping with [1], we take into consideration a KM for a CSC specified on an index set $\chi \subseteq R^d$

$$\hat{y} = \sum_{i=1}^{p} \beta_i f_i(\mathbf{x}) + z(\mathbf{x}), \qquad (1)$$

where $\boldsymbol{\beta} = (\beta_1, \dots, \beta_p)$ is a vector of unknown regression coefficients and $f(.)$ is a known function. In this case, it is assumed that the random process has a Gaussian process (GP), $Z(.) \sim GP(0, \sigma^2 V)$ where $\sigma^2 V$ is the covariance matrix, σ^2 is the overall variance, and V is the matrix of correlations [13]. We take the power exponential family as one of the many potential covariance functions, and it is represented by [14].

$$Cov(\mathbf{x}_i - \mathbf{x}_j) = \sigma^2 \exp\left(-\sum_k^p \theta_i \left|x_{ik} - x_{jk}\right|^{\alpha}\right) \quad (2)$$

where $\theta_i \geq 0$ and $0 < \alpha \leq 2$ for all k. The correlation function is

$$Cov(\mathbf{x}_i - \mathbf{x}_j) = \exp\left(-\sum_k^p \theta_i \left|x_{ik} - x_{jk}\right|^{\alpha}\right) \quad (3)$$

In this work, we set $\alpha = 2$.

With data gathered at the observation locations $\mathbf{X} = (\mathbf{x_1}, \dots, \mathbf{x_n})$, we estimate the model equation (1) and its parameters using the maximum likelihood estimation (MLE) approach [9]. As $y(x)$ is assumed as a GP with mean $\mathbf{F}\boldsymbol{\beta}$ and covariance $\sigma^2 V$, the likelihood function of $y(\mathbf{x})$ is [15]

$$L(\boldsymbol{y}, \boldsymbol{\beta}, \sigma^2,\ \boldsymbol{R}) = \frac{1}{\hat{\sigma}^2\sqrt{|\boldsymbol{R}|(2\pi)^n}} \exp\left(-\frac{1}{2\hat{\sigma}^2}\left(\boldsymbol{y} - \mathbf{F}\widehat{\boldsymbol{\beta}}\right)^{-1} \boldsymbol{R}^{-1}\left(\boldsymbol{y} - \mathbf{F}\widehat{\boldsymbol{\beta}}\right)\right), \quad (4)$$

where the design matrix is $\mathbf{F}$, R is the correlation matrix between the observation locations $\boldsymbol{R}(\mathbf{x}_i - \mathbf{x}_j)$. The likelihood function does not result in a closed-form solution, necessitating a numerical optimization. To create an accurate prediction model, we require effective MLE searching software. The linear model is then updated using the MLEs of the parameters to predict $\boldsymbol{y}(\mathbf{x})$. Using MLE, the estimated values, $\widehat{\boldsymbol{\beta}}$ and $\widehat{\boldsymbol{\theta}}$ of parameters $\boldsymbol{\beta}$ and $\boldsymbol{\theta}$, the empirical best linear unbiased prediction is provided by

$$\hat{y}(\mathbf{x}^*) = \mathbf{f}(\mathbf{x}^*)^T\widehat{\boldsymbol{\beta}} + \mathbf{r}^T\mathbf{R}^{-1}(\mathbf{y} - \boldsymbol{F}\widehat{\boldsymbol{\beta}}) \quad (5)$$

where $\boldsymbol{y}$ is the data gathered at design locations, $\mathbf{r}$ is the correlation between $\mathbf{x}^*$ and $\mathbf{x}$, and $f(\mathbf{x}^*)$ is known linear regression function.

When there is only one constant in the mean function, the KM is referred to as the ordinary KM; when there are several known variables in the mean function, it is referred to as the universal KM [16]. When using KMs, it is typical to assume that the mean function has a straightforward shape as $h(\mathbf{x}) = 1$ or $h(\mathbf{x}) = c(1, \mathbf{x})$. Clearly, such a straightforward assumption will typically be incorrect for the complex KM with uncorrelated errors. On the other side, choosing a mean function that is too complicated (overfitting) might also reduce the KM's accuracy. This is what drives the adoption of KM comparison as a method for deciding on the best mean function. Thus, choosing a suitable mean function is essential in creating a precise KM. We shall concentrate on the variable selection of the mean function in this work. To determine the influence of the chosen variables, we take into consideration several mean function options and then compare them using some measures.

This study aims to perform model selection and compare the choice with other models that are based on different mean functions for building KMs via some measures. The structure of this study is as follows: In Section 2, we review some techniques that are used for variable selection in building KMs for complex CSCs. Some validation measures for KMs are presented in Section 4. In Section 5, KMs that are based on different variable selection methods are applied to illustrative examples of CSCs. Finally, the conclusion is presented in Section 6.

2. Model Selection Methodology

A multiple regression model frequently contains one or more factors that may not be related to the response variable. By including such pointless variables, the resulting model becomes overly complex. Unfortunately, it can be time-consuming to manually go through and evaluate regression models. Fortunately, there are several methods for automatically selecting variables, or choosing those that produce the best regression results.

2.1 Stepwise Selection

One regression technique that uses an automated process to choose the predictor variables is stepwise regression. A variable chosen from a set of explanatory variables is considered for either addition or subtraction in each ongoing step, depending on a predetermined criterion determined by a variety of tests. To identify the final set of variables for use with the prediction model, a sequence of tests is employed. Stepwise Regression can be achieved by Forward Selection and Backward Selection.

2.1.1 Forward Stepwise Selection (FSS)

FSS starts with a KM that has 0 predictors and gradually adds each prediction until the KM contains all of the predictors. Specifically, the variable that improves the fit of the KM the most at each phase is added to the model [17]. The following are the three steps in the FSS procedure:

1. Suppose M_0 is he KM with no predictors. For each observation, this model only predicts the sample mean.
2. For $r = 0, \dots, p - 1$
   - All $r - p$ KMs that add one extra predictor to the ones in M_r are taken into consideration.
   - Select the top KM from this $p - k$ KMs and designate it, M_{r+1}.
3. Using AIC or BIC, choose the single best KM from among. $M_0, \dots, M_r$.

2.1.2 Backward Stepwise Selection (BSS)

An effective substitute for best subset selection is BSS. It starts with the full model with all p predictors and repeatedly eliminates the least valuable predictor, one at a time [17]. The BSS procedure can be achieved by the following three steps:

1. Let M_0 represents the comprehensive model, which includes all p predictors.
2. For $r = p,\ p-1, \dots ,1$
- For a total of $r-1$ predictors, all r KMs that include all but one of the predictors in M_r are considered.
- Select the top KM from this k KMs and denote it M_{r-1}.
3. Using AIC or BIC, choose the single best KM from among $M_0, \dots, M_r$.

The creation of a low-cost metamodel utilizing the aforementioned KM is one goal of computer experiments. A strong prediction model should be created for this objective. Hence, we will build the following models with different choices of the mean function and compare the results based on the Akaike Information Criteria (AIC) and Bayesian Information Criteria (BIC) to see the effects of the choice of the mean function. The considered KMs that are built based on different variable selection methods are described as follows:

- Kriging Model 1: $m(\mathbf{x}) = 0$.
- Kriging Model 2: $m(\mathbf{x}) = \text{constant}$ (only intercept)
- Kriging Model 3: $m(\mathbf{x}) = c(1, \mathbf{x})$
- Kriging Model 4: forward selection for a first-order linear model using AIC.
- Kriging Model 5: backward selection for a first-order linear model using AIC.
- Kriging Model 6: forward selection for a first-order linear model using BIC.
- Kriging Model 7: backward selection for a first-order linear model using BIC.

3. Validating Kriging Models

The KMs are statistical models with certain assumptions. As a result, it is essential to employ measures to assess KM behavior and determine whether or not the used assumptions were sound. In this study, we validate KMs using two measures. The root relative squared error (RRSE) and the relative absolute error (RAE) which are based on the differences between the true values of the CSC and the KM predictions. The RRSE is given by [18]

$$RRSE = \sqrt{\frac{\sum_{i=1}^{n}(\hat{y}_i - y_i)^2}{\sum_{i=1}^{n}|\bar{y}_i - y_i|^2}} \tag{6}$$

whereas the RAE is given by

$$RAE = \frac{\sum_{i=1}^{n}|\hat{y}_i - y_i|}{\sum_{i=1}^{n}|\bar{y}_i - y_i|} \tag{7}$$

where the mean of the CSC's true values is $\bar{y}$ [19]. The RRSE and the RAE reveal the disparity between predicted values and the actual value of the CSC. As $RRSE, RAE \longrightarrow 0$, the predictions of KMs will be accurate.

4. Examples

In this section, three functions are considered as examples of a CSC to see how well the presented KMs performed based on different types of mean functions. The first function is the Hartmann function, which has 6 dimensions; the second function is the Dette and Pepelyshev function which has 8 dimensions, and finally, the third function is the Piston Simulation function, which has 7 dimensions.

4.1 Hartmann Function

The Hartmann function is a 6-dimensional model that is presented by [20]. The output is given by the equation below, where the input is in the range [0 , 1],

$$f(\mathbf{x}) = \sum_{i=1}^{4} \alpha_{ij} \exp\left(-\sum_{i=1}^{4} A_{ij}\left(x_i - P_{ij}\right)\right) \tag{8}$$

We now compare the KMs models built based on different forms of the mean function based on RRSE and RAE. We generated several sets of the design points by the maximin Latin hypercube design (MLHD), $(n = 5p, 10p, 15p)$, where p is the dimension of the target function. The MLHD is a popular design proposed by [21]. Thus, we have three sets $(n = 30, 60, 90)$, of the design points. Then, the outputs $\boldsymbol{y}$, of the Hartmann function were obtained at these sets of points. The variables range of the Hartmann function was transformed to be in $[0,1]^6$.

The MLE method, equation 4, was used for estimating the KM parameters. Conditional on the estimated parameters, KMs were built. We used the power exponential correlation function equation 3 with $\alpha_k = 2$. Then, a set of $m = 2p = 12$ observations was generated by MLHD to validate the KMs. Table 1 shows the selected variables in the KMs for different variable selection methods.

Table 1: Selected prediction KMs for different variable selection methods for the Hartmann function.

n	Model	Mean Function	Selected Variables
	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
30	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6$

	KM_4	Forward based on AIC	x_1, x_2, x_4, x_5, x_6
	KM_5	Backward based on AIC	x_1, x_2, x_4, x_5, x_6
	KM_6	Forward based on BIC	x_2, x_5
	KM_7	Backward based on BIC	x_2, x_5
60	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6$
	KM_4	Forward based on AIC	x_1, x_2, x_5
	KM_5	Backward based on AIC	x_1, x_2, x_5
	KM_6	Forward based on BIC	x_1, x_2
	KM_7	Backward based on BIC	x_1, x_2
90	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6$
	KM_4	Forward based on AIC	x_1, x_4, x_5
	KM_5	Backward based on AIC	x_1, x_4, x_5
	KM_6	Forward based on BIC	x_1, x_5
	KM_7	Backward based on BIC	x_1, x_5

We can see from Table 2 that the Selected Variables using Forward and Backward methods based on BIC are only x_1, x_5. In contrast, the Selected Variables using Forward and Backward methods based on AIC are x_1, x_2, x_4, x_5, x_6 when $n = 30$. Then, they have become $x_1,\ x_4, x_5$ when $n = 60$ and 90. Table 2 presents the results of RRSE and RAE of KMs for the Hartmann function built by different forms of the mean function.

Table 2: RRSE and RAE for KMs of the Hartmann function.

n	Model	RRSE	RAE
30	KM_1	2.2459	1.9787
	KM_2	3.0059	2.3662
	KM_3	2.6030	2.1362
	KM_4	**1.7596**	**1.6160**
	KM_5	**1.7596**	**1.6160**
	KM_6	2.4036	2.5840
	KM_7	2.4036	2.5840
60	KM_1	1.5181	1.0143
	KM_2	1.8959	1.3570
	KM_3	**1.3408**	**1.0050**
	KM_4	2.4201	2.2181
	KM_5	2.4201	2.2181
	KM_6	1.9089	1.6011
	KM_7	1.9089	1.6011
90	KM_1	1.3677	1.2276
	KM_2	2.9750	1.9948
	KM_3	1.3255	1.1812
	KM_4	**1.1402**	**1.1369**
	KM_5	**1.1402**	**1.1369**
	KM_6	2.1405	1.9223
	KM_7	2.1405	1.9223

We can see from Table 4 that $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ are the best as their RRSE and RAE values are the smallest when $n = 30$. In contrast, $\boldsymbol{M_2}$ does not work well as its RRSE and RAE are high. For $n = 60$, the $\boldsymbol{M_3}$ has the smallest values of RRSE and RAE. However, when we increase the number of design points to $n = 90$, the $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ become better than $\boldsymbol{M_3}$ as their RRSE and RAE values become smaller than those of the $\boldsymbol{M_3}$.

4.2 Dette and Pepelyshev Function

The Dette and Pepelyshev function is an eight-dimensional model, and it is presented by [22]. The output is given by the equation below, where the input is in the range [0 , 1].

$$f(\mathbf{x}) = 4(x_1 - 2 + 8x_2 - 8x_2^2)^2 + (3 - 4x_2)^2 + 16\sqrt{x_3 + 1} + (3x_3 - 1)^2 \sum_{i=4}^{8} i \ln\left(1 + \sum_{j=3}^{i} x_j\right) \qquad (9)$$

where $\mathbf{x}$ is the input vector at which to evaluate and $f(\mathbf{x})$ is the function output evaluated at $\mathbf{x}$.

We generated several sets of the design points by the MLHD, $(n = 5p, 10p, 15p)$, where p is the number of variables in the target function. Thus, we have three sets $(n = 40, 80, 120)$ of the design points. Then, the outputs y, of the Dette and Pepelyshev function were obtained at these sets of points. Then, the variable range of the Dette and Pepelyshev function was transformed to be in $[0,1]^8$.

The MLE method was used to estimate KM parameters. Conditional on the estimated parameters, KMs were built based on different sets of design points. We used the power exponential correlation function equation 3 with $\alpha_k = 2$. Then, a set of $m = 2p = 16$ observations, generated by MLHD to validate the KMs. Table 3 shows the selected prediction KMs for different variable selection methods.

Table 3: Selected prediction KMs for different variable selection methods for the Dette and Pepelyshev function.

n	Model	Mean Function	Selected Variables
30	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	x_2, x_3, x_4, x_5
	KM_5	Backward based on AIC	x_2, x_3, x_4, x_5
	KM_6	Forward based on BIC	x_3
	KM_7	Backward based on BIC	x_3, x_5, x_8
60	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	x_2, x_3, x_4, x_5
	KM_5	Backward based on AIC	x_2, x_3, x_4, x_5
	KM_6	Forward based on BIC	x_2, x_3, x_4, x_5
	KM_7	Backward based on BIC	x_2, x_3, x_4, x_5
90	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	$x_1, x_2, x_3, x_4, x_5, x_6, x_8$
	KM_5	Backward based on AIC	$x_1, x_2, x_3, x_4, x_5, x_6, x_8$
	KM_6	Forward based on BIC	$x_1, x_2, x_3, x_4, x_5, x_6$
	KM_7	Backward based on BIC	$x_1, x_2, x_3, x_4, x_5, x_6$

We can see from Table 4 that the Selected Variables based on BIC are only x_3 using the Forward method and x_3, x_5, x_8 Backward method when $n = 30$. In contrast, the Selected Variables using Forward and Backward methods based on AIC are x_2, x_3, x_4, x_5. Then, for both the Forward and Backward methods, they have become $x_1, x_2, x_3, x_4, x_5, x_6$ when $n = 90$. Table 4 presents the results of RRSE and RAE of KMs for Dette and Pepelyshev built by different forms of the mean function.

Table 4: RRSE and RAE for KMs of the Dette and Pepelyshev function.

n	Model	RRSE	RAE
30	KM_1	0.7440	0.6216
	KM_2	0.5224	0.5391
	KM_3	0.4208	0.4009
	KM_4	**0.3478**	**0.3756**
	KM_5	**0.3478**	**0.3756**
	KM_6	0.5434	0.6529
	KM_7	2.4036	1.4922

60	KM_1	0.6122	0.6400
	KM_2	0.5870	0.5815
	KM_3	0.6180	0.5838
	KM_4	**0.5040**	**0.5461**
	KM_5	**0.5040**	**0.5461**
	KM_6	**0.5040**	**0.5461**
	KM_7	**0.5040**	**0.5461**
90	KM_1	0.6809	0.5417
	KM_2	0.5637	0.4813
	KM_3	0.5006	0.4999
	KM_4	**0.3203**	**0.3033**
	KM_5	**0.3203**	**0.3033**
	KM_6	0.3600	0.3194
	KM_7	0.3600	0.3194

We can see from Table 4 that M_7 does not work well as its RRSE and RAE are the highest when $n = 40$. In contrast, $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ are the best as their RRSE and RAE values are the smallest. For $n = 80$, the $\boldsymbol{M_4}, \boldsymbol{M_5}, \boldsymbol{M_6}$, and $\boldsymbol{M_7}$ are the best, and they have similar values of RRSE and RAE. However, when we increase the number of design points, the $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ become better than $\boldsymbol{M_6}$ and $\boldsymbol{M_7}$ as their RRSE and RAE values become smaller than those of the $\boldsymbol{M_6}$ and $\boldsymbol{M_7}$.

4.3 Piston Simulation Function

The Piston Simulation function was created by [23] to mimic the action of a piston inside a cylinder. The response is the cycle time, which the piston requires to achieve a single cycle in seconds.

$$C(\mathbf{x}) = 2\pi \sqrt{\frac{M}{k + S^2 \frac{P_0 V_0 T_a}{T_0 \; V^2}}} \tag{10}$$

where $V = \frac{S}{2k}\left(A^2 + 4k\frac{P_0 V_0}{T_0}T_a - A\right)$ and $A = P_0 S + 19.62M - \frac{kV_0}{S}$. The input variables are: piston weight (kg) $M \in [30, 60]$, spring coefficient $\frac{N}{m}$ $k \in [1000, 5000]$, piston surface area (m^2) $S \in [0.005, 0.020]$, atmospheric pressure $\left(\frac{N}{m^2}\right)$ $P_0 \in [90000, 110000]$, initial gas volume (m^3) $V_0 \in [0.002, 0.010]$, ambient temperature (K) $T_a \in [290, 296]$ and filling gas temperature (K) $T_0 \in [340, 360]$.

We also generated several sets of the design points by the MLHD equation 6, $(n = 5p, 10p, 15p)$, where p is the dimension of the target function. Thus, we have three sets $(n = 35, 70, 105)$ of the design points. Then, the outputs y, of the Piston Simulation Function were obtained at these sets of points. The variables range of the Piston Simulation Function was transformed to be in $[0,1]^7$.

The MLE method equation was used to estimate the KM parameters. Conditional on the estimated parameters, KMs were built based on different sets of design points. We used the power exponential correlation function equation 3 with $\alpha_k = 2$. Then, a set of $m = 2p = 14$ observations was generated by MLHD to validate the KMs. Table 5 shows the selected variables in the KMs for different variable selection methods.

Table 5: Selected prediction KMs for different variable selection methods for the Piston Simulation function.

n	Model	Mean Function	Selected Variables
30	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	x_1, x_2, x_3, x_4
	KM_5	Backward based on AIC	x_1, x_2, x_3, x_4
	KM_6	Forward based on BIC	x_1, x_2, x_3
	KM_7	Backward based on BIC	x_1, x_2, x_3
60	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	x_1, x_2, x_3, x_4, x_5
	KM_5	Backward based on AIC	x_1, x_2, x_3, x_4, x_5
	KM_6	Forward based on BIC	x_1, x_2, x_3, x_4, x_5

	KM_7	Backward based on BIC	x_1, x_2, x_3, x_4, x_5
90	KM_1	Zero mean	0
	KM_2	Constant mean	only intercept
	KM_3	Linear Mean	$x_1, x_2, x_3, x_4, x_5, x_6, x_7, x_8$
	KM_4	Forward based on AIC	x_1, x_2, x_3, x_4
	KM_5	Backward based on AIC	x_1, x_2, x_3, x_4
	KM_6	Forward based on BIC	x_1, x_2, x_3, x_4
	KM_7	Backward based on BIC	x_1, x_2, x_3, x_4

We can see from Table 6 that the Selected Variables based on BIC are only x_1, x_2, x_3 using Forward and Backward methods when $n = 30$. On the other hand, the Selected Variables using Forward and Backward methods based on AIC are x_1, x_2, x_3, x_4. Then, for both Forward and Backward methods based on BIC and AIC, the Selected Variables are the same when $n = 60$ and 90. Table 6 presents the results of RRSE and RAE of KMs for the Piston Simulation function built by different forms of the mean function.

Table 6: RRSE and RAE for KMs of the Piston Simulation function.

n	Model	RRSE	RAE
30	KM_1	0.3903	0.2895
	KM_2	0.3767	0.3005
	KM_3	0.3504	0.3240
	KM_4	**0.2769**	**0.2817**
	KM_5	**0.2769**	**0.2817**
	KM_6	0.5371	0.4352
	KM_7	0.5371	0.4352
60	KM_1	1.6812	1.5418
	KM_2	3.5211	3.1671
	KM_3	0.4336	0.4060
	KM_4	**0.0738**	**0.0785**
	KM_5	**0.0738**	**0.0785**
	KM_6	**0.0738**	**0.0785**
	KM_7	**0.0738**	**0.0785**
90	KM_1	0.0657	0.0583
	KM_2	0.0638	0.0507
	KM_3	0.0650	0.0523
	KM_4	**0.1196**	**0.1100**
	KM_5	**0.1196**	**0.1100**
	KM_6	**0.1196**	**0.1100**
	KM_7	**0.1196**	**0.1100**

We can notice from Table 6 that $\boldsymbol{M_6}$ and $\boldsymbol{M_7}$ do not work well as their RRSE and RAE are the highest when $n = 35$. In contrast, $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ are the best as their RRSE and RAE values are the smallest. For $n = 70$ and $n = 105$, the $\boldsymbol{M_4}$, $\boldsymbol{M_5}$, $\boldsymbol{M_6}$ and $\boldsymbol{M_7}$ have the smallest values of RRSE and RAE and, they work similarly, as their RRSE and RAE values are the same.
Therefore, based on the RRSE and RAE values, the results of the selected KMs for the Hartmann function, Dette and Pepelyshev Function, and Piston Simulation Function are similar. In general, the best models are $\boldsymbol{M_4}$ and $\boldsymbol{M_5}$ which are Forward and Backward selection based on AIC and BIC.

5. Conclusion

In this work, the Kriging models were reviewed for analyzing complex computer simulation codes (CSCs). The CSCs can have many variables, so identifying variables that influence the responses is necessary. Thus, in this work, we performed variable selections for several forms of the mean function of the KM. The comparison is achieved by several measures which examine the KM performance.

Based on the results of the selected KMs that are applied to the examples, the performance of the Forward and Backward based on AIC are the best among the other models. Increasing the number of design points is important. The sample size can have a great effect on the performance of the KMs when the dimension of the CSCs is high. Moreover, the values of the RRSE and RAE measures become better as well as similar for the different proposed KMs.

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7. Conflict of Interest

The authors declare no conflict of interest.

8. Funding

The authors declare that they have no known competing financial interests in this pape

9. References

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اختيار انموذج دالة الوسط في نماذج كريكنك

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المستخلص:

تُستخدم نماذج كريكنك في العديد من التخصصات العلمية لدراسة سلوك الأنظمة الفيزيائية المكلفة من الناحية العملية. في انموذج كريكنك (KM)، يفترض ان متغير الاستجابة لدالة المحاكاة الحاسوبية (CSC) له عملية كاوسية (GP) . من اجل اكتشاف المتغيرات التي تؤثر على متغير الاستجابة، يُعد اختيار مجموعة من المتغيرات أو إنشاء نموذج انحدار مُخفَّض عملية بالغة الأهمية. حيث انه يمكن لاختيار بعض المتغيرات أن يمنع الإفراط في الملاءمة أو عدم الملاءمة في التنبؤات لبيانات المحاكاة الحاسوبية. يوجد دراسات قليلة جدا حول اختيار المتغيرات في نماذج كريكنك كانموذج بديل عن دالة المحاكاة الحاسوبية. في هذا العمل، تم إجراء اختيار للمتغيرات وفقا لطريقة الاختيار الامامي والاختيار العكسي لبناء انموذج جيد بين KM. تم مقارنة نتائج اختيار النموذج المقترح من حيث دقة التنبؤ مع نماذج أخرى تستند إلى أشكال مختلفة من دالة المتوسط. تمت المقارنة بين KM من خلال بعض المقاييس التي تختبر أداء KMs. حسب النتائج التي تم الحصول عليها، اداء طريقة الاختيار الامامي والعكسي بالاعتماد على معيار AIC كان الافضل. تم تطبيق نماذج كريكنك التي تم بنائها بالاعتماد على طريقة اختيار المتغيرات على العديد من دوال المحاكاة الحاسوبية.