A New Type of ξ-Open Sets Based on Operations

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ABSTRACT

The aim of this paper is to introduce a new type of ξ-open sets in topological spaces which is called ξ_γ-open sets and we study some of their basic properties and characteristics.

Keywords: Open sets , ξ-Space.

نوع جديد من ξ – المجموعات المفتوحة على أساس العمليات

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الملخص

الهدف من هذا البحث هو دراسة نوع جديد من المجموعات المفتوحة من النمط ξ في الفضاءات التوبولوجية والتي سميت بالمجموعات المفتوحة من النمط ξ_γ وتم دراسة بعض صفات وخواص هذه المجموعة.

الكلمات المفتاحية: المجموعات المفتوحة ، الفضاء – ξ

1. Introduction

Ogata [9], introduced the concept of an operation on a topology, then after authors defined some other types of sets such as γ-open [9], γ-semi-open [6], γ-pre semi-open [6] and γ-β-open [1] sets in a topological space by using operations. In [4] the concept of ξ-open set in a topological space is introduced and studied.

The purpose of this paper, is to introduce a new class of ξ-open sets namely ξ_γ-open sets and establish basic properties and relationships with other types of sets, also we define the notions of ξ_γ-neighbourhood, ξ_γ-derived, ξ_γ-closure and ξ_γ-interior of a set and give some of their properties which are mostly analogous to those properties of open sets. Throughout this paper, (X, τ) or(briefly, X) mean a topological space on which no separation axioms are assumed unless explicitly stated. Let A be a subset of a topological space X, Cl(A) and Int(A) are denoted respectively the closure and interior of A.

2. Preliminaries.

We start this section by introducing some definitions and results concerning sets and spaces which will be used later.

Definition 2.1. A subset A of a space (X, τ) is called:

1) semi-open [7], if $A \subseteq Cl(Int(A))$.
2) regular open [2], if $A = Int(Cl(A))$.

The complement of semi-open (resp., regular open, preopen and α-open) set is said to be semi-closed (resp., regular closed, preclosed and α-closed).

Definition 2.2. [4] An open subset U of a space X is called ξ-open if for each $x \in U$, there exists a semi-closed set F such that $x \in F \subseteq U$. The family of all ξ-open subsets of a topological space (X, τ) is denoted by $\xi O(X, \tau)$ or (briefly $\xi O(X)$). The complement of each ξ-open set is called ξ-closed set. The family of all ξ-closed subsets of a topological space (X, τ) is denoted by $\xi C(X, \tau)$ or (briefly $\xi C(X)$).

Definition 2.3. [5] Let (X, τ) be a topological space. An operation γ on the topology τ is a mapping from τ into power set P(X) such that $V \subseteq \gamma(V)$ for each $V \in \tau$, where $\gamma(V)$ denotes the value of γ at V.

Definition 2.4. [8]

1) A subset A of a topological space (X, τ) is called γ-open set if for each $x \in A$ there exists an open set U such that $x \in U$ and $\gamma(U) \subseteq A$. Clearly $\tau_\gamma \subseteq \tau$.
   Complements of γ-open sets are called γ-closed.
2) The point $x \in X$ is in the γ-closure of a set $A \subseteq X$, if $\gamma(U) \cap A \neq \phi$, for each open set U containing x. The γ-closure of a set A is denoted by $Cl_\gamma(A)$.
3) Let (X, τ) be a topological space and A be subset of X, then $\tau_\gamma\text{-}Cl(A) = \cap\{ F: A \subseteq F, X \setminus F \in \tau_\gamma\}$.

Definition 2.5. [11] Let (X, τ) be a topological space and A be subset of X, then $\tau_\gamma\text{-}Int(A) = \cup\{ U : U$ is γ-open set and $U \subseteq A\}$.

Definition 2.6. [1] Let (X, τ) be a topological space with an operation γ on τ:

1) The γ-derived set of A is defined by $\{x$: for every γ-open set U containing x, $U \cap (A \setminus \{x\}) \neq \phi\}$
2) The γ-boundary of A is defined as $\tau_\gamma\text{-}Cl(A) \cap \tau_\gamma\text{-}Cl(X \setminus A)$.

Definition 2.7. [4] Let (X, τ) be a topological space and $A \subseteq X$, then:

1) ξ-interior of A is the union of all ξ-open sets contained in A.
2) ξ-closure of A is the intersection of all ξ-closed sets containing A.

Lemma 2.8. [4]

1) Let (Y, τ_Y) be a subspace of (X, τ). If $F \in SC(X, \tau)$ and $F \subseteq Y$, then $F \in SC(Y, \tau_Y)$.
2) Let (Y, τ_Y) be a subspace of (X, τ). If $F \in SC(Y, \tau_Y)$ and $Y \in SC(X, \tau)$, then $F \in SC(X, \tau)$.

Lemma 2.9 [4]

1) Let Y be a regular open subspace of a space X. If $G \in \xi O(Y)$, then $G \in \xi O(X)$.
2) Let Y be a subspace of a space X and $Y \in SC(X)$. If $G \in \xi O(X)$ and $G \subseteq Y$, then $G \in \xi O(Y)$.

3. ξ_γ-Open Sets

In this section, a new class of ξ-open sets called ξ_γ-open sets in topological spaces is introduced. We define γ to be a mapping on $\xi O(X)$ into P(X) and we say that $\gamma: \xi O(X) \rightarrow P(X)$ is an ξ-operation on $\xi O(X)$ if $V \subseteq \gamma(V)$, for each $V \in \xi O(X)$.

Definition 3.1 A subset A of a space X is called ξ_γ-open if for each point x$\in$ A, there exist an ξ-open set U such that x$\in$ U $\subseteq \gamma$(U) $\subseteq$ A.

The family of all ξ_γ-open subset of a topological space (X, τ) is denoted by ξ_γO(X, τ) or (briefly ξ_γO(X)).

A subset B of a space X is called ξ_γ-closed if X \B is ξ_γ-open. The family of all ξ_γ-closed subsets of a topological space (X, τ) is denoted by ξ_γC(X, τ) or (briefly ξ_γC(X)).

Remark 3.2 From the definition of the operation γ, it is clear that γ(X)=X for any ξ-operation γ. For competence, it is assumed that $\gamma(\phi)= \phi$ for any ξ-operation γ.

Remark 3.3 It is clear from the definition that every ξ_γ-open subset of a space X is ξ-open, but the converse is not true in general as shown in the following example:

Example 3.5. Consider X = {a, b, c, d} with the topology
τ = {ϕ, X, {c}, {a, b}, {a, b, c}}. Define an ξ-operation γ by

$$\gamma(A) = \begin{cases} A & if \quad a \in A \\ X & if \quad a \notin A \end{cases}$$

Then {c} is open and ξ-open but {c}$\notin \xi_\gamma$O(X).

Proposition 3.6. Every ξ_γ-open set of a space X is γ-open.

Proof. Let A be ξ_γ-open in a topological space (X, τ), then for each point x$\in$ A, there exists an ξ-open set U such that x$\in$ U $\subseteq \gamma$(U) $\subseteq$ A. Since every ξ-open set is open, this implies that A is a γ-open set.

The following example shows that the converse of the above proposition is not true in general.

Example 3.7 Consider X = {a, b, c} with topology τ = {ϕ, X, {a}}. Define an ξ-operation γ by γ(A) =A, for any subset A of X. Then, {a} is γ-open set but not ξ-open set. Hence, it is not ξ_γ-open.

The following result shows that any union of ξ_γ-open sets in a topological space (X, τ) is ξ_γ-open.

Proposition 3.8 Let $\{A_\lambda\}_{\lambda\in\Delta}$ be a collection of ξ_γ-open sets in a topological space (X, τ). Then, $\bigcup_{\lambda\in\Delta} A_\lambda$ is ξ_γ-open.

Proof. Let x$\in \bigcup_{\lambda\in\Delta} A_\lambda$, then x$\in A_\lambda$ for some $\lambda \in \Delta$. Since, A_λ is an ξ_γ-open set, then there exists an ξ_γ-open set U containing x and γ(U) $\subseteq A_\lambda \subseteq \bigcup_{\lambda\in\Delta} A_\lambda$. Therefore, $\bigcup_{\lambda\in\Delta} A_\lambda$ is an ξ_γ-open set in a topological space (X, τ).

The following example shows that the intersection of two ξ_γ-open sets need not be an ξ_γ-open set.

Example 3.9 Consider X = {a, b, c} with discrete topology on X. Define an ξ-operation γ by

$$\gamma(A)=\begin{cases}\{a,b\} & if \quad A=\{a\}\ or\ \{b\}\\ A & otherwise\end{cases}$$

Let A ={a, b} and B ={b, c}, it is clear that A and B are ξ_γ-open sets, but A∩B={b} is not ξ_γ-open set.

From the above example, we notice that the family of all ξ_γ-open subsets of a space X is a supratopology and need not be a topology in general.

Proposition 3.10 The set A is ξ_γ-open in the space (X, τ) if and only if for each x∈ A, there exists an ξ-open set B such that x∈ B ⊆ A.

Proof. Suppose that A is an ξ_γ-open set in the space (X, τ). Then, for each x∈ A, put B=A is an ξ-open set such that x∈ B ⊆ A.

Conversely, suppose that for each x∈ A, there exists an ξ-open set B_x such that x∈ B_x ⊆ A, thus A= $\cup B_x$ where $B_x \in \xi_\gamma O(X)$ for each x∈ A. Therefore, A is ξ_γ-open set.

Definition 3.11 Let (X, τ) be a topological space. A mapping γ : ξO(X) → P(X) is said to be :

1) ξ-identity on ξO(X) if γ(A) = A for all A∈ ξO(X).
2) ξ-monotone on ξO(X) if for all A, B ∈ ξO(X), A⊆ B implies γ(A) ⊆γ(B).
3) ξ-idempotent on ξO(X) if γ(γ(A)) = γ(A) for all A∈ ξO(X).
4) ξ-additive on ξO(X) if γ(A∪B) = γ(A) ∪ γ(B) for all A, B∈ ξO(X).

If $\cup_{i\in I}\gamma(A_i) \subseteq \gamma(\cup_{i\in I}A_i)$ for any collection $\{A_i\}_{i\in I} \subseteq \xi O(X)$, then γ is said to be ξ-subadditive on ξO(X).

Proposition 3.12. Let γ be an ξ-operation. Then, γ is ξ-monotone on ξO(X) if and only if γ is subadditive on ξO(X).

Proof. Let γ be ξ-monotone on ξO(X) and let $\{A_i\}_{i\in I} \subseteq \xi O(X)$. Then, for each i ∈I, $\gamma(A_i) \subseteq \gamma(\cup_{i\in I}A_i)$ and thus $\cup_{i\in I}\gamma(A_i) \subseteq \gamma(\cup_{i\in I}A_i)$. Therefore, γ is ξ- subadditive on ξO(X).

Conversely, if γ is subadditive on ξO(X), and A, B∈ ξO(X) with A⊆ B, then γ(A) ⊆ γ(A) ∪ γ(B) ⊆ γ(A∪B) = γ(B). Thus, γ is ξ-monotone on ξO(X).

The following result shows that if γ is ξ-monotone, then the family of ξ_γ-open sets is a topology on X.

Proposition 3.13 If γ is ξ-monotone , then the family of ξ_γ-open sets is a topology on X.

Proof. Clearly ϕ, X $\in \xi_\gamma O(X)$ and by Proposition3.8, the union of any family ξ_γ-open sets is ξ_γ-open set. To complete the proof, it is enough to show that the finite intersection of ξ_γ-open sets is an ξ_γ-open set. Let A and B be two ξ_γ-open sets and let x ∈ A ∩ B, then x ∈ A and x ∈ B, so there exists ξ_γ-open sets namely U and V such that x ∈ U ⊆ γ(U) ⊆ A and x ∈ V ⊆ γ(V) ⊆ B, since U and V are ξ-open sets then U ∩ V is ξ-open, but U ∩ V⊆U and U ∩ V ⊆ V, but γ is ξ-monotone operation, therefore γ(U ∩V) ⊆ γ(U) ∩ γ(V) ⊆ A ∩ B. Thus, A ∩ B is an ξ_γ-open set. This completes the proof.

Proposition 3.14 Let Y be a semi-closed subspace of a space X. If A ∈ $\xi_\gamma O(X, \tau)$ and A ⊆ Y, then A ∈ $\xi_\gamma O(Y, \tau_Y)$, where γ is ξ-identity on ξO(Y).

Proof. Let $A \in \xi_\gamma O(X, \tau)$, then $A \in \xi O(X, \tau)$ and for each $x \in A$ there exists an ξ-open set U in X such that $x \in U \subseteq \gamma(U) \subseteq A$. Since, $A \in \xi O(X, \tau)$ and $A \subseteq Y$, where Y is semi-closed in X, then by Proposition 2.14, $U \in \xi_{\gamma/}O(Y, \tau_Y)$. Hence, $A \in \xi_{\gamma/}O(Y, \tau_Y)$.

Proposition 3.15 Let Y be a regular open subspace of a space (X, τ) and γ is an ξ-identity on ξO(X). If $A \in \xi_\gamma O(Y, \tau_Y)$ and $Y \in \xi O(X, \tau)$, then $A \in \xi_\gamma O(X, \tau)$.

Proof. Let $A \in \xi_{\gamma/}O(Y, \tau_Y)$, then $A \in \xi O(Y, \tau_Y)$ and for each $x \in A$ there exists an ξ-open set U in Y such that $x \in U \subseteq \gamma/\ (U) \subseteq A$. Since, $Y \in \xi O(X, \tau)$ and $A \in \xi O(Y, \tau_Y)$, then by Proposition 2.13, $U \in \xi O(X, \tau)$. Hence, $A \in \xi_\gamma O(X, \tau)$.

4. Other Properties of ξ_γ-Open Sets

In this section, we define and study some properties of ξ_γ-neighbourhood of a point, ξ_γ-derived, ξ_γ-closure and ξ_γ-interior of sets via ξ_γ-open sets.

Definition 4.1 Let (X, τ) be a topological space and $x \in X$, then a subset N of X is said to be ξ_γ-neighbourhood of x, if there exists an ξ_γ-open set U in X such that $x \in U \subseteq N$.

Proposition 4.2 Let (X, τ) be a topological space. A subset A of X is ξ_γ-open if and only if it is an ξ_γ-neighbourhood of each its points.

Proof. Let $A \subseteq X$ be an ξ_γ-open set. Since, for every $x \in A$, $x \in A \subseteq A$ and A is ξ_γ-open, then A is an ξ_γ-neighbourhood of each its points.
Conversely, suppose that A is an ξ_γ-neighbourhood of each its points. Then, for each $x \in A$, there exists $B_x \in \xi_\gamma O(X)$ such that $B_x \subseteq A$. Then, $A = \cup\{ B_x : x \in A\}$. Since, each B_x is ξ_γ-open, It follows that A is an ξ_γ-open set.

Definition 4.3 Let (X, τ) be a topological space with an operation γ on ξO(X). A point $x \in X$ is said to be ξ_γ-limit point of a set A if for each ξ_γ-open set U containing x , then $U \cap (A \setminus \{x\}) \neq \phi$. The set of all ξ_γ-limit points of A is called ξ_γ-derived set of A and denoted by ξ_γ-D(A).

Proposition 4.5 Let A and B be subsets of a space X. If $A \subseteq B$, then $\xi_\gamma\text{-}D(A) \subseteq \xi_\gamma\text{-}D(B)$.

Proof. Obvious.

Some properties of ξ_γ-derived sets are stated in the following proposition.

Proposition 4.6 Let A and B be any two subsets of a space X, and γ be an operation on ξO(X). Then, we have the following properties:

1) $\xi_\gamma\text{-}D(\phi) = \phi$.
2) If $x \in \xi_\gamma\text{-}D(A)$, then $x \in \xi_\gamma\text{-}D(A \setminus \{x\})$.
3) $\xi_\gamma\text{-}D(A) \cup \xi_\gamma\text{-}D(B) \subseteq \xi_\gamma\text{-}D(A \cup B)$.
4) $\xi_\gamma\text{-}D(A \cap B) \subseteq \xi_\gamma\text{-}D(A) \cap \xi_\gamma\text{-}D(B)$.
5) $\xi_\gamma\text{-}D(\xi_\gamma\text{-}D(A)) \setminus A \subseteq \xi_\gamma\text{-}D(A)$.
6) $\xi_\gamma\text{-}D(A \cup \xi_\gamma\text{-}D(A) \subseteq A \cup \xi_\gamma\text{-}D(A)$.

Proof. Straightforward.

In general, the equalities of (3), (4) and (6) of the above proposition do not hold, as is shown in the following examples.

Example 4.7 Consider X = {a, b, c} with discrete topology on X. Define an operation γ on ξO(X) by

$$\gamma(A)=\begin{cases} A & if \quad A=\{b\}\ or\ \{a,b\}\ \ or\ \ \{a,c\} \\ X & otherwise \end{cases}$$

Now, if A = {a, b} and B = {a, c}, then ξ_γ-D(A) = {c}, ξ_γ-D(B) = {c} and ξ_γ-D(A ∪ B) ={a, c}, where A ∪ B= X, this implies that ξ_γ-D(A) ∪ ξ_γ-D(B) ≠ ξ_γ-D(A ∪ B).

Example 4.8 Consider X = {a, b, c, d} with the topology τ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}. Define an operation γ on ξO(X) by.

$$\gamma(A)=\begin{cases} A & if \quad b\in A \\ X & if \quad b\notin A \end{cases}$$

Now, if we let A = {a, b} and B ={c, d}, then ξ_γ-D(A) = {a, c, d}, ξ_γ-D(B) = {d}, hence ξ_γ-D(A) ∩ ξ_γ-D(B)= {d}, but ξ_γ-D(A ∩ B) = ϕ, where A ∩ B = ϕ, this implies that ξ_γ-D(A ∩ B) ≠ ξ_γ-D(A) ∩ ξ_γ-D(B). Also ξD(A) ={d}, therefore ξ_γ-D(A) ⊄ ξD(A).

Definition 4.9 Let A be a subset of a topological space (X, τ) and γ be an operation on ξO(X). The intersection of all ξ_γ-closed sets containing A is called the ξ_γ-closure of A and denoted by ξ_γ-Cl(A).

Here, we introduce some properties of ξ_γ-closure of the sets.

Proposition 4.10 Let (X, τ) be a topological space and γ be an operation on ξO(X). For any subsets A and B of X, we have the following:

1) A ⊆ ξ_γ-Cl(A).
2) ξ_γ-Cl(A) is an ξ_γ-closed set in X.
3) A is an ξ_γ-closed set if and only if A=ξ_γ-Cl(A).
4) ξ_γ-Cl(ϕ) = ϕ and ξ_γ-Cl(X) = X.
5) ξ_γ-Cl(A) ∪ ξ_γ-Cl(B) ⊆ ξ_γ-Cl(A ∪ B).
6) ξ_γ-Cl(A∩ B) ⊆ ξ_γ-Cl(A) ∩ ξ_γ-Cl(B).

Proof. They are obvious.

In general, the equalities of (5) and (6) of the above proposition does not hold, as is shown in the following examples:

Example 4.11 Consider X = {a, b, c} with discrete topology on X. Define an operation γ on ξO(X) by

$$\gamma(A)=\begin{cases} A & if \quad A=\{a,b\}\ \ or\ \ \{a,c\} \\ X & otherwise \end{cases}$$

Then, ξ_γO(X) = {ϕ, X, {a, b}, {a, c}}. Now, if we let A = {b} and B = {c}, then ξ_γ-CL(A) = A, ξ_γ-D(B) = B and ξ_γ-Cl(A ∪ B) =X, where A ∪ B= {b, c}, this implies that ξ_γ-Cl(A) ∪ ξ_γ-Cl(B) ≠ ξ_γ-Cl(A ∪ B).

Example 4.12 Consider X = {a, b, c, d} with the topology τ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}. Define an operation γ on ξO(X) by.

$$\gamma(A)=\begin{cases}A & if \quad b\in A\\ X & if \quad b\notin A\end{cases}$$

It is clear that ξ_γ-O(X) = {ϕ, X, {b}, {a, b}, {b, c}, {a, b, c}}. Now, if we let A = {c} and B ={d}, then ξ_γ-Cl(A) = {c, d}and ξ_γ-Cl(B) = {d},
hence ξ_γ-Cl(A) ∩ ξ_γ-Cl(B)= {d}, but ξ_γ-Cl(A ∩ B) = ϕ, where A ∩ B = ϕ, this implies that ξ_γ-CL(A ∩ B) ≠ ξ_γ-Cl(A) ∩ ξ_γ-Cl(B).
Now, if we let A= {b}, we see that ξCl(A) = {b, d}, but ξ_γ-Cl(A) = X. Hence, ξ_γ-Cl(A) ⊄ ξCl(A).

Proposition 4.13 A subset A of a topological space X is an ξ_γ-closed set if and only if it contains the set of its ξ_γ-limit points.

Proof. Assume that A is an ξ_γ-closed set and if possible that x is an ξ_γ-limit point of A which belongs to X \ A, then X \A is an ξ_γ-open set containing the ξ_γ-limit point of A, therefore, A ∩ (X\A) ≠ ϕ, which is contradiction.
Conversely, assume that A is containing the set of its ξ_γ-limit points. For each x ∈X\A, there exists an ξ_γ-open set U containing x such that A ∩ U = ϕ, implies that x ∈U ⊆ X\A, so by Proposition 3.10, X\A is an ξ_γ-open set hence, A is an ξ_γ-closed set.

Proposition 4.14 Let A be a subset of a topological space (X, τ) and γ be an ξ-operation. Then, x ∈ ξ_γCl(A) if and only if for every ξ_γ-open set V of X containing x, A ∩ V ≠ ϕ.

Proof. Let x ∈ ξ_γCl(A) and suppose that A ∩ V = ϕ, for some ξ_γ-open set V of X containing x. Then, (X\V) is ξ_γ-closed and A ⊆ (X\V), thus ξ_γCl(A) ⊆ (X\V). But, this implies that x ∈ (X\V) which is contradiction. Therefore,
A ∩ V ≠ ϕ.
Conversely, Let A ⊆ X and x ∈X such that for each ξ_γ-open set V of X containing x, A ∩ V ≠ ϕ. If x ∉ ξ_γCL(A), there exists an ξ_γ-closed set F such that A ⊆ F. Then, (X\F) is an ξ_γ-open set with x ∈ (X\F), and thus (X\ F)∩A ≠ ϕ, which is a contradiction.
The proof of the following two results is obvious.

Proposition 4.15 Let A be a subset of a topological space (X, τ) and γ be an ξ-operation on ξO(X). Then, ξ_γCl(A) = A ∪ξ_γD(A).

Proposition 4.16 If A and B are subsets of a space X with A ⊆ B. Then, ξ_γCl(A) ⊆ ξ_γCl(B).

Definition 4.17 Let A be a subset of a topological space (X, τ) and γ be an operation on ξO(X). The union of all ξ_γ-open sets contained in A is called the ξ_γ-Interior of A and denoted by ξ_γ-Int(A).

Here, we introduce some properties of ξ_γ-Interior of the sets.

Proposition 4.18 Let (X, τ) be a topological space and γ be an operation on ξO(X). For any subsets A and B of X, we have the following:
1) ξ_γ-Int(A) is an ξ_γ-open set in X.
2) A is ξ_γ-open if and only if A=ξ_γ-Int(A).
3) ξ_γ-Int(ξ_γ-IntA)) =ξ_γ-Int(A).

4) ξ_γ-Int(ϕ) = ϕ and ξ_γ-Int(X) = X.
5) ξ_γ-Int(A) $\subseteq$ A.
6) If A $\subseteq$ B, then ξ_γ-Int(A) $\subseteq$ ξ_γ-Int(B).
7) ξ_γ-Int(A) $\cup$ ξ_γ-Int(B) $\subseteq$ ξ_γ-Int(A $\cup$ B).
8) ξ_γ-Int(A$\cap$ B) $\subseteq$ ξ_γ-Int(A) $\cap$ ξ_γ-Int(B).

Proof. Straightforward.

In general, the equalities of (7) and (8) of the above proposition do not hold, as is shown in the following examples:

Example 4.19 Consider X = {a, b, c, d} with the topology τ = {ϕ, X, {a}, {b}, {c}, {a, b}, {a, c}, {b, c}, {a, b, c}}. Define an ξ-operation γ by.

$$\gamma(A) = \begin{cases} A & if \quad b \in A \\ X & if \quad b \notin A \end{cases}$$

It is clear that $\xi\gamma$-O(X) = {ϕ, X, {b}, {a, b}, {b, c}, {a, b, c}}. Now, if we let A = {a} and B ={b}, then $\xi\gamma$-Int(A) = ϕ and $\xi\gamma$-Int(B) = {b}, hence $\xi\gamma$-Int(A) $\cup$ $\xi\gamma$-Int(B)= {b}, but $\xi\gamma$-int(A$\cup$ B) = {a, b}, where A $\cup$ B = {a, b}, this implies that $\xi\gamma$-Int(A $\cup$ B) $\neq$ $\xi\gamma$-Int(A) $\cup$ $\xi\gamma$-Int(B).

Example 4.20 Consider X = {a, b, c} with discrete topology on X. Define an ξ-operation γ on ξO(X) by

$$\gamma(A) = \begin{cases} A & if \quad A = \{a,b\} \quad or \quad \{a,c\} \\ X & otherwise \end{cases}$$

Then, ξ_γO(X) = {ϕ, X, {a, b}, {a, c}}. Now, if we let A = {a, b} and B = {a, c}, then ξ_γ-Int(A) = {a, b}and ξ_γ-Int(B) = {a, c}, therefore ξ_γ-Int(A) $\cap$$\xi_\gamma$-Int(B)= {a}, but ξ_γ-Int(A $\cap$ B) = ϕ, where A $\cap$ B= {a}, this implies that ξ_γ-Int(A) $\cap$ ξ_γ-Int(B) $\neq$ ξ_γ-Int(A $\cap$ B).

The following two results can be easily proved.

Proposition 4.21 For any subset A of a topological space X, ξ_γ-Int(A) $\subseteq$ ξInt(A) $\subseteq$ Int(A).

Proposition 4.22 Let A be any subset of a topological space X, and γ be an operation on ξO(X). Then, ξ_γ-Int(A) = A \ ξ_γ-D(X \ A).

REFERENCES

[1] Basu, C. K., Afsan, B. M. U. and Ghosh, M. K., (2009), A class of functions and separation axioms with respect to an operation. Hacettepe *journal of Mathematics and Statistics*, 38 (2), 103-118.

[2] Dugundji, J., (1966), **Topology**, Allyn and Bacon Inc., Boston, .

[3] El-Deeb, S. N., Hasanein, I. A., Mashhour, A. S. and Noiri, T., (1983), On P-regular spaces, *Ball. Math. Sci. Math. R. S. Rounmanie*, 27 (4), 311-315.

[4] Hasan M. Haji, (2010), On Some Types of Continuity, Separation Axioms and Dimension Functions, *ph.D. Thesis, Duhok Univ., Duhok*, .

[5] Kasahara, S., (1979), Operation compact spaces, *Math. Japonica*, 24 (1), 97-105.

[6] Krishnan, G. S. and Balachandran, K., (2006), On γ-semi-open sets in topological space, *Bull. Cal. Math. Soc*., 98 (6), 517-530.

[7] Liven, N., (1963), Semi-open sets and semi-continuity in topological spaces, *Amer. Math. Monthly*, 70 (1), 36-41.

[8] Moiz K., (1997), Weak forms of continuity, compactness and connectedness, *ph.D. Thesis, Multan Univ., Pakistan*,.

[9] Ogata, H., (1991), Operation on topological spaces and associated topology, *Math. Japonica*, 36 (1), 175-184.

[10] Reilly, I. L. and Vamanmurthy, M.K., (1985), On α-contiuity in topological spaces, *Acta Math. Hungar*., 45 (1-2) , 27-32.

[11] Sia sundra Krishnan, G., (2003), A new class of semi-open sets in a topological space, *Proc. NCMCM, Allied Publishers, new Delhi*, 305-311.