On Representation Theorem for Algebras with Three Commuting Involutions

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ABSTRACT

Algebras with three commuting involutions are represented as commutants of one-generated ◇ – □ – ○ subalgebras of algebras of vector-space endomorphisms where ◇ – □ and ○ are involutions of a prefixed type.

Keywords: Algebras, commuting involutions.

حول نظرية التمثيل للجبريات مع ثلاث تشابكات إبدالية

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الملخص

تم تقديم الجبريات مع ثلاثة تشابكات ابدالية كمولد واحد للجبور الجزئية ◇ – □ – ○ من جبور فضاء متجه التطبيقات الخطية حيث ◇ – □ – ○ رموز التشابكات المثبتة المذكورة آنفاً.

الكلمات المفتاحية: الجبريات، تشابكات ابدالية.

Introduction and Preliminaries

Throughout this paper *(k, –)* denotes a field with an involution and the terminology of algebra and algebra involution is relative to *(k, –)*. A systematic study of representation theory for algebras with involutions was given in [6] by Quebbemann and he proved that (involutive unital finite-dimensional algebras can be represented as commutants of one-generated self-adjoint subalgebras of algebras of vector-space endomorphisms) and later the representation theory for algebras with involutions has been extend to algebras with two commuting involutions by Cabrera and Mohammed see [1].

We begin by summarizing some definitions and fundamental concepts. An involution * in an algebra A is a mapping $a \rightarrow a^*$ of A into it self satisfying $(a+b)^* = a^* + b^*$, $(\alpha a)^* = \bar{\alpha}\, a^*$ (where – denote the conjecate of complex number), $(ab)^*=b^*a^*$ and $a^{**}=a$ for all a, b in A and α in k see[1]. A subalgebra of A globally invariant by * is called a

$*$ – subalgebra. If B is a $*$ – subalgebra of A, then its centralizer in A given by

$$\{ a \in A : ab=ba \text{ for all } b \text{ in } B \},$$

is also a $*$ – subalgebra of A see [1].

Involutive algebras can be constructed from the consideration of nondegenerate hermitian spaces. Recall that, for Σ in k satisfying $\Sigma\overline{\Sigma}=1$, a nondegenerate Σ–hermitian form in a vector – space M over k is a mapping $<.,.>$ from M x M into k satisfying

$< m_1 + m_2, m' > = < m_1, m' > + < m_2, m' >,\ < \alpha m, m' > = \alpha < m, m' >$

$< m , m' > = \Sigma \overline{< m', m >}$

for all m_1, m_2, m, m' in M and α in k, and $< m, m' > = 0$ for all m' implies $m=0$ see [1]. If M has finite dimension, then the algebra $\mathrm{End}_k(M)$ of all endomorphisms of M with the adjoint involution $F \rightarrow F^{\diamond}$ given by

$< F(m), m' > = < m, F^{\diamond}(m') >$

for all m, m' in M see [1].

If $(A, *, \#, \delta)$, $(B, \diamond, \square, \bigcirc)$ are algebras with three commuting involutions, an isomorphism between $(A, *, \#, \delta)$ and $(B, \diamond, \square, \bigcirc)$ is an algebra isomorphism ϕ from A onto B satisfying $\phi(a^*) = \phi(a)^{\diamond}$, $\phi(a^{\#}) = \phi(a)^{\mathbf{G}}$ and $\phi(a^{\delta}) = \phi(a)^{F}$ for all a in A. In this case $(A, *, \#, \delta)$ and $(B, \diamond, \square, \bigcirc)$ are said to be isomorphic see [1].

Our main result is the following :

Theorem 1. Let $(A, *, \#, \delta)$ be a unital finite-dimensional algebra with commuting involutions over $(k, -)$ and let $\Sigma, \Sigma', \Sigma''$ in k such that $\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = \Sigma''\overline{\Sigma}'' = 1$. Then there exist a finite-dimensional vector space w over k, a nondegenerate Σ-hermitian form $<.,.>$, a nondegenerate Σ'-hermitian form $[.,.]$, a nondegenerate Σ''-hermitian form $[.,.]$, and F in $\mathrm{End}_k(w)$ such that $(A, *, \#, \delta)$ is isomorphic to the centralizer of the $\diamond$– $\square$ – $\bigcirc$– subalgebra of $\mathrm{End}_k(w)$ generated by F, where $\diamond$, $\square$ and $\bigcirc$ are adjoint involutions in $\mathrm{End}_k(w)$ determined by $<.,.>$, $[.,.]$ and $(.,.)$, respectively.

We will follow the lines of the following lemma : Let $(A, *, \#)$ be a unital finite-dimensional algebra with commuting involutions over $(K, -)$ and let Σ, Σ' in K such that $\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = 1$. Then there exist a finite-dimensional vector space W over K, a nondegenerate Σ-hermitian form $<.,.>$ in W, a nodegenerate Σ'-hermitian from $[.,.]$ in W, and F in $End_K(W)$ such that $(A, *, \#)$ is isomorphic to the centralizer of the $\diamond$– $\square$– subalgebra of $End_K(W)$ generated by F, where $\diamond$ and $\square$ are adjoint involutions in $End_K(W)$ determined by $< .,. >$ and $[.,.]$ respectively.

Proof : see [1, Theorem 1]

The first part of the proof consists in finding a nondegenerate $\Sigma - \Sigma' - \Sigma''$-hermitian space w_o over k such that $(A, *, \#, \delta)$ is embedded into $End_k(w_o)$ in such a way that w_o is a balanced A-module (that is, $A = End_B(w_o)$ if $B = End_A(w_o)$). Our construction involves the three commuting involutions of A and consists in a convenient triple of the representation used in [1].

Theorem 2. let $(A, *, \#, \delta)$ be a unital finite-dimensional algebra with commuting involutions over $(k, -)$ and let $\Sigma, \Sigma', \Sigma''$ in k such that $\Sigma\bar{\Sigma} = \Sigma'\bar{\Sigma}' = \Sigma''\bar{\Sigma}'' = 1$. Then there exists $(w_o, <. , . >, [. , .], (. , .))$, where w_o is a finite-dimensional vector space over k which is a balanced left A-module (in fact, w_o contains A as a direct summand) and $<. , .>$, $[. , .]$, $(. , .)$ are nondegenerate Σ-hermitian, Σ'-hermitian and Σ"-hermitian forms in w_o, respectively, in such a way that the associated representation of A in w_o becomes an isomorphism of algebras with three involutions of $(A, *, \#, \delta)$ into $(End_k(w_o), \diamondsuit, \square, \bigcirc)$, where $\diamondsuit$, $\square$ and $\bigcirc$ are adjoint involutions in $End_k(w_o)$ determined by $<. , .>$, $[. , .]$ and $(. , .)$, respectively.

Proof. Consider the vector space $w_o := U_1 \oplus U_2 \oplus U_3 \oplus U_4 \oplus U_5 \oplus U_6$, where $U_1 = U_3 = U_5 = A$ and $U_2 = U_4 = U_6 = Hom_k(A, k)$. Endow w_o with the structure of faithful left A-module given by :
$a(x_1, f_1, x_2, f_2, x_3, f_3) := (ax_1, f_1 o L_{a*}, a^{*\#}x_2, f_2 o L_{a\#}, a^{*\#\delta}x_3, f_3 o L_{a\delta})$ for all a in A and $(x_1, f_1, x_2, f_2, x_3, f_3)$ in w_o. The mapping $<. , .>$ from w_o x w_o in to k defined by

$$<(x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)> := \underline{f_1(y_1) + f_2(y_2) + f_3(y_3)} + \Sigma(g_1(x_1) + g_2(x_2) + g_3(x_3))$$

is a nondegenerate Σ-hermitian form satisfying

$$<a(x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)> = <(x_1, f_1, x_2, f_2, x_3, f_3), a^*(y_1, g_1, y_2, g_2, y_3, g_3)>,$$

and therefore the representation of A on w_o becomes an isomorphism of involutive algebras from $(A, *)$ into $(End_k(w_o), \diamondsuit)$, where $\diamondsuit$ denotes the adjoint involution with respect to $<. , .>$. Furthermore, the mapping $[. , .]$ from w_o x w_o into k defined by

$$[(x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)] := \underline{f_1(y_2) + f_2(y_3) + f_3(y_1)} + \Sigma(g_1(x_2) + g_2(x_3) + g_3(x_1))$$

is a nondegenerate Σ'-hermitian form satisfying

$$[a(x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)] = [(x_1, f_1, x_2, f_2, x_3, f_3), a^{\#}(y_1, g_1, y_2, g_2, y_3, g_3)],$$

and so the representation of A on w_o also becomes an isomorphism of involutive algebras from (A, #) into (End_k (w_o), $\boldsymbol{G}$), where $\boldsymbol{G}$ denotes the adjoint involution with respect to [.,.]. furthermore, the mapping (. , .) from w_o x w_o into k defined by

$((x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)) :=$
$f_1(y_3) + f_2(y_1) + f_3(y_2) + \Sigma'' (g_1(x_3) + g_2(x_1) + g_3(x_2))$
is a nondegenerate Σ"-hermitian form satisfying
$[a (x_1, f_1, x_2, f_2, x_3, f_3), (y_1, g_1, y_2, g_2, y_3, g_3)] =$
$[(x_1, f_1, x_2, f_2, x_3, f_3), a^{\delta}(y_1, g_1, y_2, g_2, y_3, g_3)]$,

and so the representation of A on w_o also becomes an isomorphism of involutive algebras from (A, δ) into (End_k(w_o), $\boldsymbol{F}$), where $\boldsymbol{F}$ denotes the adjoint involution with respect to (. , .). Therefore, the representation of A on w_o is an isomorphism of algebras with three involutions. Since w_o contains the "regular" A-module A as a direct summand, it is balanced (see [4, P. 451]).■

Remark 1. The involutions $\diamond$, $\square$ and $\bigcirc$ in End_k(w_o) obtained in the above proof are not necessarily commuting. Since $w_o = U_1 \oplus U_2 \oplus U_3 \oplus U_4 \oplus U_5 \oplus U_6$ we can represent each T in End_k (w_o) as a 6x6 homomorphism matrix.

$$T = \begin{pmatrix} T_{11} & T_{12} & T_{13} & T_{14} & T_{15} & T_{16} \\ T_{21} & T_{22} & T_{23} & T_{24} & T_{25} & T_{26} \\ T_{31} & T_{32} & T_{33} & T_{34} & T_{35} & T_{36} \\ T_{41} & T_{42} & T_{43} & T_{44} & T_{45} & T_{46} \\ T_{51} & T_{52} & T_{53} & T_{54} & T_{55} & T_{56} \\ T_{61} & T_{62} & T_{63} & T_{64} & T_{65} & T_{66} \end{pmatrix}$$

Where $T_{ij} \in Hom_k (U_j, U_i)$ for $i , j \in \{ 1, 2, 3, 4, 5, 6 \}$. It is easy to verify that

$$T^{\diamond} = \begin{pmatrix} T'_{22} & \bar{\Sigma}T'_{12} & T'_{42} & \bar{\Sigma}T'_{32} & T'_{62} & \bar{\Sigma}T'_{52} \\ \Sigma T'_{21} & T'_{11} & \Sigma T'_{41} & T'_{31} & \Sigma T'_{61} & T'_{51} \\ T'_{24} & \bar{\Sigma}T'_{14} & T'_{44} & \bar{\Sigma}T'_{34} & T'_{64} & \bar{\Sigma}T'_{54} \\ \Sigma T'_{23} & T'_{13} & \Sigma T'_{43} & T'_{33} & \Sigma T'_{63} & T'_{53} \\ T'_{26} & \bar{\Sigma}T'_{16} & T'_{46} & \bar{\Sigma}T'_{36} & T'_{66} & \bar{\Sigma}T'_{56} \\ \Sigma T'_{25} & T'_{15} & \Sigma T'_{45} & T'_{35} & \Sigma T'_{65} & T'_{55} \end{pmatrix} ,$$

$$T^{\square} = \begin{pmatrix} T'_{66} & \overline{\Sigma'}\, T'_{56} & T'_{46} & \overline{\Sigma'}\, T'_{36} & T'_{26} & \overline{\Sigma'}\, T'_{16} \\ \Sigma' T'_{65} & T'_{55} & \Sigma' T'_{45} & T'_{35} & \Sigma' T'_{25} & T'_{15} \\ T'_{64} & \overline{\Sigma'}\, T'_{54} & T'_{44} & \overline{\Sigma'}\, T'_{34} & T'_{24} & \overline{\Sigma'}\, T'_{14} \\ \Sigma' T'_{63} & T'_{53} & \Sigma' T'_{43} & T'_{33} & \Sigma' T'_{23} & T'_{13} \\ T'_{62} & \overline{\Sigma'}\, T'_{52} & T'_{42} & \overline{\Sigma'}\, T'_{32} & T'_{22} & \overline{\Sigma'}\, T'_{12} \\ \Sigma' T'_{61} & T'_{51} & \Sigma' T'_{41} & T'_{31} & \Sigma' T'_{21} & T'_{11} \end{pmatrix}$$

And

$$T^{\bigcirc} = \begin{pmatrix} T'_{44} & \overline{\Sigma''}\, T'_{14} & T'_{64} & \overline{\Sigma''}\, T'_{34} & T'_{24} & \overline{\Sigma''} T'_{54} \\ \Sigma'' T'_{41} & T'_{11} & \Sigma' T'_{61} & T'_{31} & \Sigma'' T'_{21} & T'_{51} \\ T'_{46} & \overline{\Sigma''} T'_{16} & T'_{66} & \overline{\Sigma''} T'_{36} & T'_{26} & \overline{\Sigma''} T'_{56} \\ \Sigma'' T'_{43} & T'_{13} & \Sigma'' T'_{63} & T'_{33} & \Sigma'' T'_{23} & T'_{53} \\ T'_{42} & \overline{\Sigma''} T'_{12} & T'_{62} & \overline{\Sigma''} T'_{32} & T'_{22} & \overline{\Sigma''} T'_{52} \\ \Sigma'' T'_{45} & T'_{15} & \Sigma'' T'_{65} & T'_{35} & \Sigma'' T'_{25} & T'_{55} \end{pmatrix}$$

Therefore

$$T^{\diamond\square} = \begin{pmatrix} T'_{55} & \overline{\Sigma'}\, \Sigma T'_{56} & T'_{53} & \overline{\Sigma'}\, \Sigma T'_{54} & T'_{51} & \overline{\Sigma'}\, \Sigma T'_{52} \\ \Sigma' \overline{\Sigma} T'_{65} & T'_{66} & \Sigma' \overline{\Sigma} T'_{63} & T'_{64} & \Sigma' \overline{\Sigma} T'_{61} & T'_{62} \\ T'_{35} & \overline{\Sigma'}\, \Sigma T'_{36} & T'_{33} & \overline{\Sigma'}\, \Sigma T'_{34} & T'_{31} & \overline{\Sigma'}\, \Sigma T'_{32} \\ \Sigma' \overline{\Sigma} T'_{45} & T'_{46} & \Sigma' \overline{\Sigma} T'_{43} & T'_{44} & \Sigma' \overline{\Sigma} T'_{41} & T'_{42} \\ T'_{15} & \overline{\Sigma'}\, \Sigma T'_{16} & T'_{13} & \overline{\Sigma'}\, \Sigma T'_{14} & T'_{11} & \overline{\Sigma'}\, \Sigma T'_{12} \\ \Sigma' \overline{\Sigma} T'_{25} & T'_{26} & \Sigma' \overline{\Sigma} T'_{23} & T'_{24} & \Sigma' \overline{\Sigma} T'_{21} & T'_{22} \end{pmatrix}$$

$$T^{\square\diamond} = \begin{pmatrix} T'_{55} & \overline{\Sigma} \Sigma' T'_{56} & T'_{53} & \overline{\Sigma} \Sigma' T'_{54} & T'_{51} & \overline{\Sigma} \Sigma' T'_{52} \\ \Sigma \overline{\Sigma'} T'_{65} & T'_{66} & \Sigma \overline{\Sigma'} T'_{63} & T'_{64} & \Sigma \overline{\Sigma'} T'_{61} & T'_{62} \\ T'_{35} & \overline{\Sigma} \Sigma' T'_{36} & T'_{33} & \overline{\Sigma} \Sigma' T'_{34} & T'_{31} & \overline{\Sigma} \Sigma' T'_{32} \\ \Sigma \overline{\Sigma'} T'_{45} & T'_{46} & \Sigma \overline{\Sigma'} T'_{43} & T'_{44} & \Sigma \overline{\Sigma'} T'_{41} & T'_{42} \\ T'_{15} & \overline{\Sigma} \Sigma' T'_{16} & T'_{13} & \overline{\Sigma} \Sigma' T'_{14} & T'_{11} & \overline{\Sigma} \Sigma' T'_{12} \\ \Sigma \overline{\Sigma'} T'_{25} & T'_{26} & \Sigma \overline{\Sigma'} T'_{23} & T'_{24} & \Sigma \overline{\Sigma'} T'_{21} & T'_{22} \end{pmatrix},$$

And

$$T^{\Diamond\bigcirc}=\begin{pmatrix} T'_{55} & \overline{\Sigma''}\Sigma T'_{56} & T'_{53} & \overline{\Sigma''}\Sigma T'_{54} & T'_{51} & \overline{\Sigma''}\Sigma T'_{52} \\ \Sigma''\overline{\Sigma}\,T'_{65} & T'_{66} & \Sigma''\overline{\Sigma}\,T'_{63} & T'_{64} & \Sigma''\overline{\Sigma}\,T'_{61} & T'_{62} \\ T'_{35} & \overline{\Sigma''}\Sigma T'_{36} & T'_{33} & \overline{\Sigma''}\Sigma T'_{34} & T'_{31} & \overline{\Sigma''}\Sigma T'_{32} \\ \Sigma''\overline{\Sigma}\,T'_{45} & T'_{46} & \Sigma''\overline{\Sigma}\,T'_{43} & T'_{44} & \Sigma''\overline{\Sigma}\,T'_{41} & T'_{42} \\ T'_{15} & \overline{\Sigma''}\Sigma T'_{16} & T'_{13} & \overline{\Sigma''}\Sigma T'_{14} & T'_{11} & \overline{\Sigma''}\Sigma T'_{12} \\ \Sigma''\overline{\Sigma}\,T'_{25} & T'_{26} & \Sigma''\overline{\Sigma}\,T'_{23} & T'_{24} & \Sigma''\overline{\Sigma}\,T'_{21} & T'_{22} \end{pmatrix}$$

As a result, $\Diamond$, $\Box$ and $\bigcirc$ are commuting if and only if $\overline{\Sigma'}\,\Sigma = \overline{\Sigma}\,\Sigma' = \overline{\Sigma''}\,\Sigma = \overline{\Sigma}\,\Sigma''$, or equivalently $\Sigma^2 = \Sigma'^2 = \Sigma''^2$.

Proof of Theorem 1. let $(A, *, \#, \delta)$ be a unital finite-dimensional algebra with commuting involutions over $(k, -)$ and let Σ, Σ', Σ'' in k such that $\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma'} = \Sigma''\overline{\Sigma''} = 1$. By Theorem 2 there exists (w_o, <.,.>, [.,.] , (.,.)), where w_o is a finite-dimensional vector space over k which is a balanced left A-module and <.,.>, [.,.] and (.,.) are nondegenerate Σ-hermitian, Σ'-hermitian and Σ"-hermitian forms in w_o, respectively, in such a way the associated representation of A in w_o becomes an isomorphism from $(A, *, \#, \delta)$ into ($\mathrm{End}_k(w_o)$, $\Diamond$, $\Box$, $\bigcirc$), where $\Diamond$, $\Box$ and $\bigcirc$ are adjoint involutions in $\mathrm{End}_k(w_o)$ determined by <.,.>, [.,.] and (.,.), respectively. Let m denote the dimension of $B = \mathrm{End}_A(w_o)$. Put (w, <.,.>, [.,.], (.,.)) : = (w_o, <.,.> , [.,.] , (.,.)) $\oplus \overset{m+2}{\cdots\cdots} \oplus$ (w_o, <.,.>, [.,.], (.,.)), and consider $\mathrm{End}_k(w_o)$ embedded diagonally in $\mathrm{End}_k(w)$. By the final step of the proof of theorem 1 in [1] applied to <.,.>, [.,.] and (.,.) there exists F in $\mathrm{End}_k(w)$ such that $A=\mathrm{End}_C(w) = \mathrm{End}_D(w) = \mathrm{End}_H(w)$, where C and D (resp. H) denotes the $\Diamond$ – subalgebra and $\Box$ – subalgebra (resp. O – subalgebra) of $\mathrm{End}_K(w)$ generated by F. Let us denote by E the $\Diamond$– $\Box$ – $\bigcirc$– subalgebra of $\mathrm{End}_k(w)$ generated by F. Since $C, D, H \subseteq E$, it follows that $\mathrm{End}_E(w) \subseteq \mathrm{End}_C(w) = \mathrm{End}_D(w) = \mathrm{End}_H(w) = A$. On the other hand, A is a $\Diamond$– $\Box$ – $\bigcirc$– subalgebra of $\mathrm{End}_k(w)$ whose elements commute with F, therefore $\mathrm{End}_A(w)$ is a $\Diamond$– $\Box$ – $\bigcirc$– subalgebra of $\mathrm{End}_k(w)$ containing F, and so $E \subseteq \mathrm{End}_A(w)$. From this, $A \subseteq \mathrm{End}_E(w)$.

Remark 2. The process of representing the algebras of commuting involutions can be explained through the following diagram :

No. of involution	**Underliny vector – space of represented algebra**	**Construction of Nondegenerate form with respect to involution**	**Generator of represented algebra with involution**	**The condition on the element of the field *k*, where** $\Sigma, \overline{\Sigma}, \ldots \in k$
N=1	$w_o = U_1 \oplus U_2$	< . , . >	T ∈ $End_k(w_o)$ and T ∈ M_{2x2}(¢)	$\Sigma\overline{\Sigma} = 1$
N=2	$w_o = U_1 \oplus U_2 \oplus U_3 \oplus U_4$	< . , . > , [. , .]	T ∈ $End_k(w_o)$ and T ∈ M_{4x4}(¢)	$\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = 1$
N=3	$w_o = U_1 \oplus \ldots\ldots \oplus U_6$	<. , .> , [. , .] , (. , .)	T ∈ $End_k(w_o)$ and T ∈ M_{6x6}(¢)	$\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = \Sigma'\overline{\Sigma''} = 1$
N=4	$w_o = U_1 \oplus \ldots\ldots \oplus U_8$	<.,.> , [.,.] , (.,.) , {.,.}	T ∈ $End_k(w_o)$ and T ∈ M_{8x8}(¢)	$\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = \Sigma'\overline{\Sigma''} = \Sigma''\overline{\Sigma'''} = 1$
N=5	$w_o = U_1 \oplus \ldots\ldots \oplus U_{10}$	<.,.>, [. , .], (. , .), {. , .}, ((. , .))	T ∈ $End_k(w_o)$ and T∈M_{10x10}(¢)	$\Sigma\overline{\Sigma} = \Sigma'\overline{\Sigma}' = \Sigma'\overline{\Sigma''} = \Sigma''\overline{\Sigma'''} = \Sigma'''\overline{\Sigma''''} = 1$
. . . .	. . . .	. . . .	. . . .	. . . .

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